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Bloch–Floquet waves in flexural systems with continuous and discrete elements

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ABSTRACT

In this paper we describe the dynamic behavior of elongated multi-structured media excited by flexural harmonic waves. We examine periodic structures consisting of continuous beams and discrete resonators disposed in various arrangements. The transfer matrix approach and Bloch–Floquet conditions are implemented for the determination of different propagation and non-propagation regimes. The effects of the disposition of the elements in the unit cell and of the contrast in the physical properties of the different phases have been analyzed in detail, using representations in different spaces and selecting a proper set of non-dimensional parameters that fully characterize the structure. Coupling in series and in parallel continuous beam elements and discrete resonators, we have proposed a class of micro-structured mechanical systems capable to control wave propagation within elastic structures.

1. Introduction

Waves propagating in a homogeneous continuum are non-dispersive, since their phase and group velocities coincide. Conversely, in a structured medium (e.g. a rod, a beam or a plate) dispersion occurs due to the presence of physical boundaries. A heterogeneous medium is also characterized by stop-bands, which are intervals of frequencies at which waves decay exponentially. Heterogeneities may be represented either by an intrinsic microstructure or by structural interfaces.

Some real structures are made of modular units, equal to each other, that are joined together. Despite being finite in reality, these structures can be analyzed as infinite sequences of identical elements connected to each other ("periodic structures"), as demonstrated by Wei and Petyt (1997a,b) for beams, by Brun et al. (2011) for bridges and by Carta et al. (2014a,b) for damaged strips.

Wave propagation in periodic structures has been extensively studied. In his classical treatise Brillouin (1953) gave a unified formulation for different classes of problems. In the literature, periodic structures are labeled according to the order of the equation describing the motion of the structure. It is usually convenient to express a differential equation of order p as a system of p first-order differential equations as, for example, in Stroh (1962), Lekhnitskii (1963) and Ting (1996). The number of kinematic independent variables ($p/2$) defines the coupling at the interface between two modular units (or "unit cells"): "mono-coupled" if $p/2 = 1$, "bi-coupled" if $p/2 = 2$, and so on (see Mead, 1975a,b). Mono-coupled periodic structures (such as rods, one-dimensional lattices, etc.) were investigated by Mead (1975a), Faulkner and Hong (1985) and, more recently, by Martinsson and Movchan (2002), Brun et al. (2010) and Carta and Brun (2012). Bi-coupled periodic structures (like beams) were examined by Mead (1975b, 1996), Heckl (2002), Romeo and Luongo (2002) and Carta et al. (2014a,b). In this paper, attention is focused on bi-coupled periodic structures, with particular interest to civil engineering structures (bridges,

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pipelines, railways, etc.), micro-mechanical systems (e.g. periodic MEMS bridges) and biological systems (like neurons).

The study of linear problems in elastodynamics has been carried out with different approaches. Heckl (2002), Movchan and Slepyan (2007), Brun et al. (2011) and Movchan and Slepyan (2014) implemented a quasi-periodic Green's function, complemented in Movchan et al. (2007) and McPhedran et al. (2009) by the multipole expansion method for defects of finite size. A more direct procedure consists in solving directly the equations of motion by introducing the proper boundary conditions, as in Bigoni and Movchan (2002), Gei et al. (2009) and Brun et al. (2013). The transfer matrix method (Pestel and Leckie, 1963; Faulkner and Hong, 1985; Lekner, 1994; Castanier and Pierre, 1995; Romeo and Luongo, 2002; Brun et al., 2010) proves to be an efficient tool for increasingly complex periodic structures, in the sense that complicated unit cells can be easily decomposed into simpler sub-units while the dimension of the transfer matrix is independent of the complexity of the unit cell. In this work we will use the transfer matrix approach for continuous and discrete bi-coupled systems connected in series and in parallel.

The determination of the dispersion properties of the system and of pass- and stop-bands (i.e. the intervals of frequencies in which waves can propagate or not) can be performed in different ways. The most common approach is to use the dispersion relation, which shows the dependence of the frequency on the wavenumber or wavevector. The analysis of the invariants of the transfer matrix is useful to determine all possible propagation characteristics of a periodic structure. The identification of propagation and non-propagation zones in a physical-space, where the coordinate variables are the physical parameters of the structure, is of primary importance for the design of filtering systems. Another approach is based on the study of the eigenvalues of the characteristic (or secular) equation of the periodic structure. These four alternative techniques are outlined in Section 2 (the reader may also refer to the extensive analysis by Romeo and Luongo (2002) for a more detailed discussion of the mentioned techniques). Section 3 presents the study of wave propagation in four types of bi-coupled periodic structures, consisting of different arrangements of continuous and discrete elements (or "phases") connected either in series or in parallel. The periodic systems described in Section 3 are employed in different engineering structures. For instance, beams in

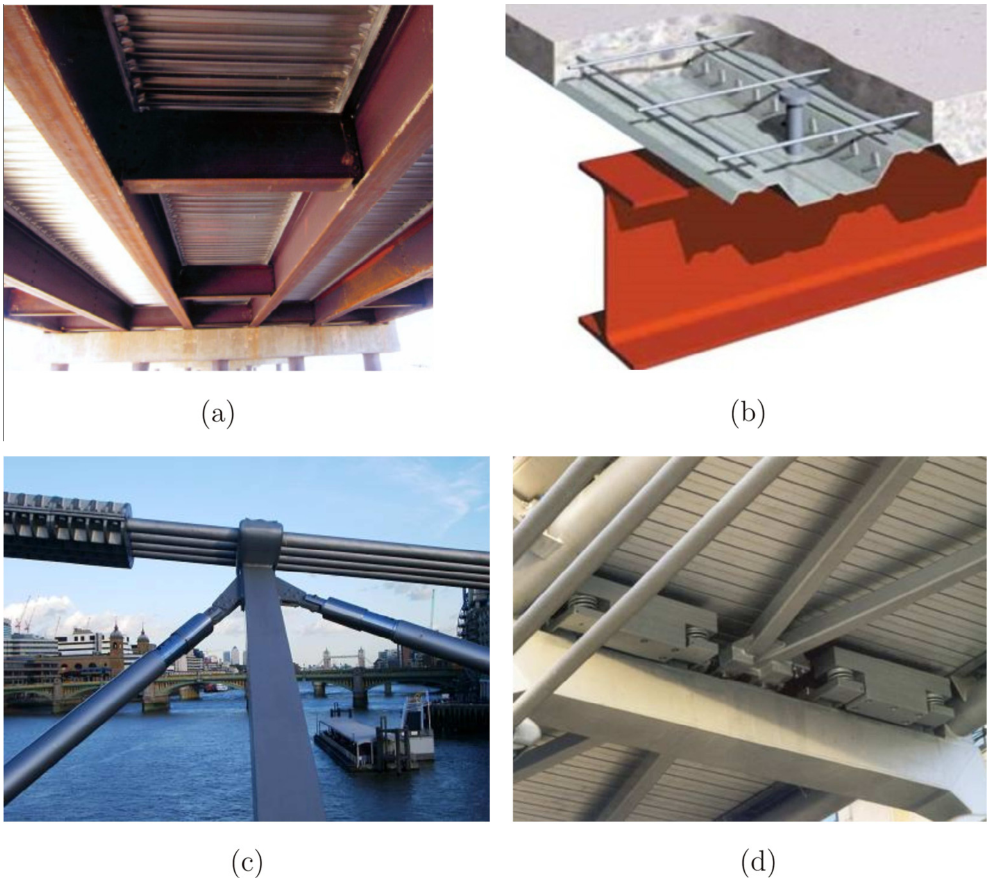

Fig. 1. (a) Steel girders of a bridge crossing Box Elder Creek in Watkins, Colorado, USA (from www.shortspansteelbridges.org, accessed on 08/09/2014); (b) detail of a composite beam consisting of a concrete slab, steel girders and shear connectors (from www.theconstructor.org, accessed on 08/09/2014); (c) viscous dampers used in the Millennium Bridge, London, UK (from www.mace.manchester.ac.uk, accessed on 09/09/2014); (d) tuned mass dampers employed in the Millennium Bridge, London, UK (from www.gerbusa.com, accessed on 09/09/2014).