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Flexible cam profile synthesis method using smoothing spline curves

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Abstract

This paper describes a synthesis method of designing flexible cam profiles by using smoothing spline curves. Our approach simultaneously considers displacement, velocity and acceleration rather than displacement alone in the design process. The inherent characteristic of the smoothing spline ensures the smoothness of cam profiles designed by the proposed method. The essential difference between the proposed method and other existing approaches is that our method allows designers to refine the cam motion by local modifications while still satisfying profile smoothness and discrete constraints. Particularly, the method can control not only the level but also the shape of the velocity and acceleration curves for better kinematical and dynamic properties. Examples show that the obtained cam profiles provide solutions for some problems in automotive engine valve trains.

Keywords: Smoothing spline; Cam profile; Cubic spline; Quintic spline; Valve train

1. Introduction

Cams are widely used in many types of machines because they make it possible to obtain an unlimited variety of motions. Many different types of cam profiles are designed and manufactured depending on a machine's requirements [1]. Because some cam follower systems such as automotive engine valve trains operate at very high speeds, cam profiles must be precisely refined to ensure the kinematical and dynamic properties [2]. Not only the displacement curve but also the velocity curve, acceleration curve and even the jerk curve must be controlled in some applications as well. There are an infinite number of ways to express a cam profile mathematically. The function of standard motions include *harmonic*, *cycloid*, *modified harmonic*, *trapezoidal*, *modified trapezoidal*, *polynomial*, etc. When the requirements are not extremely difficult, the traditional methods

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work effectively. Depending on conditions, these mathematical methods may be directly applied, modified or combined into piecewise functions [1].

Spline curves have been successfully applied in cam design for more than 20 years [3–11]. Angeles [3] seems to be the first person to employ spline curves on cam profile designs. This method was then improved by MacCarthy and Burns [4]. Spline methods are flexible enough to permit motion programs to be refined or optimized while still satisfying motion constraints. As such, spline methods have been applied to synthesize cam motions that overcome difficulties associated with the traditional approaches. Previous researches by Tsay and Huey, [5–7], Sandgren and West [8], and Yoon and Rao [9], all suggest the potential use of spline methods in cam design. However, early applications of the spline method did not aim directly to solve the general problems of cam motion synthesis. The process of solving such problems includes the task of satisfying a set of arbitrarily specified displacement constraints. In some cases, constraints on the derivatives were imposed, but the constraints were not applied in a general and systematic manner. Mermelstein and Acar [10] introduce an optimal algorithm to design cams by using piecewise polynomials. However, this method considers the jerk minimization and boundary constraints only. It does not simultaneously consider multiple design requirements, namely, requirements in displacement, velocity and acceleration. Qiu, Lin and others introduced a universal optimal approach to cam design [11]. The flexible object function in their method is an effective tool to simultaneously deal with multiple objects in either kinematical or dynamic optimizations. However, because it is based on the B-spline curves modifications of control points only affect some neighboring segments [7,8]. Accordingly, the result of this method may hold some uninvited extraordinary points, especially when local adjustments are required in the velocity and/or acceleration curves. Cam designers must often not only refine the peak values of velocity, acceleration and jerk curves but also shift the locations of those peak values away from critical regions [12]. Moreover, designers sometimes make local adjustments to reduce the pressure angle at some troublesome spots in the cam profile [13,14]. These requirements are quite important for high-speed cam follower systems such as automotive engine valve trains.

This paper proposes a new synthesis method for designing flexible cam profiles. Based on smoothing spline curves with some constraints, this approach achieves cam profiles that satisfy requirements in displacement, velocity and acceleration curves. Apart from the smoothness, which is the implicit essence of the proposed method, our method can also strictly constrain the shape of displacement, velocity and acceleration curves as well as the extreme values. This property, which is one of our aims, provides a very powerful tool in dealing with cam profiles of automotive engine valve trains.

This paper is organized as follows: In Section 2, we present the mathematical formulation of the flexible cam design method and some background information related to solving the optimization problem. Sections 3.1 and 3.2 present two algorithms using different smoothing spline methods: cubic and quintic smoothing spline curves respectively. The Lagrange multipliers and numerical solution methods of the proposed algorithms are shown in Section 3.3. For illustrative purposes, some examples are shown in Section 4. Section 5 concludes the paper.

2. Method of flexible cam design

2.1. The flexible cam design problem

Cam designers usually work with constraints on displacement, velocity, and acceleration curves. The designed cam profile must satisfy these constraints. It is assumed that Y_i, V_i, and A_i are sets of given constraints in displacement, velocity, and acceleration of a cam follower system at corresponding camshaft rotation angles x_i with $i = 0, \ldots, n$ and $x_0 < x_1 < \ldots < x_n$, respectively.

To find the curve that best fits a given set of points, the mathematical concept of "*residuals*" is introduced, which is the sum of the squares of offsets of the points from the curve. The sum of the squares is usually used instead of the absolute residual values to allow the residuals to be treated as a continuously differentiable quantity. If the designed cam profile curve is defined as $g(x)$, the constraints in displacement, velocity and acceleration can be expressed as follows.