Available online at www.sciencedirect.com
ScienceDirect
Mechanism and Machine Theory 42 (2007) 859–870
Mechanism and Machine Theory
www.elsevier.com/locate/mechmt

Analysis and dimensional synthesis of the DELTA robot for a prescribed workspace

M.A. Laribi [a,b], L. Romdhane [a,*], S. Zeghloul [b]

[a] *Laboratoire de Génie Mécanique, LAB-MA-05, Ecole Nationale d'Ingénieurs de Monastir, Monastir 5019, Tunisia*
[b] *Laboratoire de Mécanique des Solides, CNRS UMR 6610, Bd Pierre et Marie Curie, BP 30179, 86962 Futuroscope Chasseneuil cedex, France*

Received 23 September 2005; accepted 14 June 2006
Available online 10 October 2006

Abstract

This paper deals with an optimal dimensional synthesis method of the DELTA parallel robot for a prescribed workspace. The objective function is based on the mathematical concept of the power of a point with respect to bounding constraint surfaces. A genetic algorithm based method was used to solve this problem. The proposed method is simple and was shown to be effective in finding the dimensions of the DELTA robot having the smallest workspace containing a prespecified region in space. These dimensions were also determined in the case where the user defines a safety region, to avoid points in the prescribed workspace being on the boundary of the DELTA robot's actual workspace.

Keywords: DELTA parallel robot; Synthesis; Genetic algorithm; Workspace; Power of a point; Safety region

1. Introduction

The Stewart platform mechanism is a fully parallel mechanism. In the general sense, each of these mechanisms consists of two platforms that are connected by six prismatic joints acting in parallel. One of the platforms is defined as the moving platform. It has six degrees of freedom relative to the other fixed platform, which is the "base".

The analysis of this type of mechanisms has been the focus of much recent research. Stewart presented his platform in 1965 [20]. Since then, several authors [5,6] have proposed a large variety of designs.

The interest in parallel manipulators (PM's) arises from the fact that they exhibit high stiffness in nearly all configurations and a high dynamic performance. Recently, there has been a growing tendency to focus on parallel manipulators with 3-translational degree of freedom (dof) [1,4,10,11,17,18,23,24]. In this case, the mobile platform can only translate, along the three cartesian axes, with respect to the base. The DELTA robot is one of the most famous translational parallel manipulators [4,25,21].

* Corresponding author. Tel.: +216 73500244; fax: +216 73500514.
E-mail address: lotfi.romdhane@enim.rnu.tn (L. Romdhane).

0094-114X/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mechmachtheory.2006.06.012

However, as most of the authors mentioned above have pointed out, the major drawback of parallel manipulators is their limited workspace. Gosselin [7], separated the workspace, which corresponds to a six-dimensional coordinate space, in two parts: positioning and orientation workspace. He studied only the positioning workspace, i.e., the region of the three-dimensional cartesian space that can be attained by a point on the top platform when its orientation is given. A number of authors have described the workspace of a parallel mechanism by discretizing the cartesian workspace. Concerning the orientation workspace, Romdhane [19] was the first to address the problem of its determination. In the case of 3-translational degree of freedom (dof) manipulators, the workspace is limited to a region of the three-dimensional cartesian space that can be attained by a point on the mobile platform.

A more challenging problem is designing a parallel manipulator for a given workspace. This problem has been addressed by Boudreau and Gosselin [2,3], who proposed an algorithm that allows for the determination of some parameters of the parallel manipulators using a genetic algorithm method in order to obtain a workspace as close as possible to a prescribed one. Kosinska et al. [12] presented a method for the determination of the parameters of a Delta-4 manipulator, where the prescribed workspace has been given in the form of a set of points. Snyman et al. [22] propose an algorithm for designing the planar 3-RPR manipulator parameters, for a prescribed two-dimensional physically reachable output workspace. Similarly in [8] the synthesis of a 3-dof planar manipulators with prismatic joints is performed using a GA, where the architecture of a manipulator and its position and orientation with respect to the prescribed workspace were determined.

In this paper, the three translational dof DELTA robot is designed to have a specified workspace. A genetic algorithm (GA) is used to solve the optimization problem, because of its robustness and simplicity.

This paper is organized as follows: Section 2 is devoted to the kinematic analysis of the DELTA robot and the determination of its workspace. In Section 3, we carry out the formulation of the optimization problem using the genetic algorithm technique. Section 4 deals with the implementation of the proposed method followed by the obtained results. Finally, Section 5 contains some conclusions.

2. Kinematic analysis and workspace of the DELTA robot

2.1. Direct and inverse geometric analysis

The DELTA robot consists of a moving platform connected to a fixed base through three parallel kinematic chains. Each chain contains a rotational joint activated by actuators in the base platform. The motion is transmitted to the mobile platform through parallelograms formed by links and spherical joints (see Fig. 1).

We assume that all the three legs of the DELTA robot are identical in length. The geometric parameters of the DELTA robot are L_1, L_2, r_A, r_B, θ_j ($j = 1,2,3$) as defined in Fig. 1, as well as φ_{1j}, φ_{2j}, φ_{3j} ($j = 1,2,3$), the joint angles defining the configuration of each leg.

Let P be a point located on the moving platform, then the geometric model can be written as

$$X_P = \cos\theta_j(r_A + L_2\cos\varphi_{1j} + L_1\cos\varphi_{3j}\cos(\varphi_{1j} + \varphi_{2j}) - r_B) - L_1\sin\theta_j\sin\varphi_{3j} \tag{1}$$

$$Y_P = \sin\theta_j(r_A + L_2\cos\varphi_{1j} + L_1\cos\varphi_{3j}\cos(\varphi_{1j} + \varphi_{2j}) - r_B) + L_1\cos\theta_j\sin\varphi_{3j} \tag{2}$$

$$Z_P = L_2\sin\varphi_{1j} + L_1\cos\varphi_{3j}\sin(\varphi_{1j} + \varphi_{2j}) \tag{3}$$

with $j = 1,2,3$ and where $[X_P, Y_P, Z_P]$ are the coordinates of the point P in the fixed reference frame $(\overrightarrow{X}, \overrightarrow{Y}, \overrightarrow{Z})$ as shown in Fig. 1.

In order to eliminate the passive joint variables, we square and add these equations

$$[(r + L_2\cos\varphi_{1j})\cos\theta_j - X_P]^2 + [(r + L_2\cos\varphi_{1j})\sin\theta_j - Y_P]^2 + [-L_2\sin\varphi_{1j} - Z_P]^2 - L_1^2 = 0 \tag{4}$$

where $j = 1,2,3$ and $r = r_A - r_B$.

2.1.1. The direct geometric model

The direct problem is defined by (4), where the unknowns are the location of the point $P = [X_p, Y_p, Z_p]$ to be determined for given joint angles φ_{1j}, φ_{2j}, φ_{3j} ($j = 1,2,3$).