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On characteristic parameters involved in dynamic fragmentation processes

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ABSTRACT

Dynamic loadings produce high stress waves leading to the fragmentation of brittle materials such as ceramics, concrete, glass, and rocks, or ductile materials such as steels and alloys. The main mechanism used herein to explain the change of the number of fragments with strain and stress rates is an obscuration (or shielding) phenomenon associated with cracking or cavitation. A probabilistic framework, which is based upon a Poisson point process, is introduced. Nonlocal (in space and time) expressions are obtained to account for multiple crack initiations or void nucleations, and their subsequent growth. This approach allows characteristic parameters (*i.e.* size, stress, stress rate, and time) involved in the fragmentation processes to be introduced. Examples are discussed to illustrate the use of these characteristic parameters in the analysis of dynamic fragmentation of brittle materials and spallation of tantalum.

1. Introduction

Dynamic loadings induce very severe degradations in brittle and ductile materials. In brittle materials, very dense crack patterns are observed when impacted by projectiles or loaded by blast (*e.g.* see Rinehart, 1965; Kutter and Fairhurst, 1971; Shockey et al., 1974; Hornemann et al., 1984; Cagnoux, 1985; Strassburger et al., 1994). Similarly, ductile spallation is the result of nucleation and growth (and possibly coalescence) of many cavities in a given region associated with the interaction of stress waves (*e.g.* see Mott, 1947; Meyers and Aimone, 1983; Grady, 1988; Curran et al., 1993).

In many instances, the elementary damage mechanisms are studied (*i.e.* either a single propagating crack (*e.g.* see Freund, 1972; Ravi-Chandar and Knauss, 1982; Strassburger and Senf, 1995), or a single growing cavity (*e.g.* see Carroll and Holt, 1972; Glennie, 1972; Ortiz and Molinari, 1992; Molinari and Mercier, 2001)). Of the three phases of the fragmentation process, namely, initiation, propagation and coalescence of cracks, or nucleation, growth and coalescence of voids, the main emphasis is therefore put on the modeling of propagation and growth. The validation of the developed models is generally performed with comparisons to macroscopic data, which are very difficult to extract from dynamic experiments.

Probabilistic aspects associated with brittle or ductile fragmentations were recognized very early on (Mott, 1947; Shockey et al., 1974; Grady and Kipp, 1980). The Weibull law has been used as basis to describe inception mechanisms regimes (Denoual et al., 1997; Czarnota et al., 2006). However, their experimental characterization is even more difficult since it requires statistical data and possibly some information on their development during the test. The use of high speed imaging has allowed some progress to be made in the understanding of the experiment. However, in many instances, the pictures are used in a qualitative way. For brittle materials, the development of so-called edge-on-impact experiments has brought new insights into the cracking process of

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brittle materials (Strassburger et al., 1994; Riou et al., 1998). It is even possible to measure displacement fields (Bertin-Mourot et al., 1997) and then use the strain fields to validate damage models (Denoual and Hild, 2000, 2002).

One important aspect related to fragmentation studies is the interaction between growing cracks and voids and the inception of new voids (Mott, 1947) or cracks (Denoual et al., 1997). Some of the potential sites where inception may take place are inhibited because too close to growing cracks or voids. This phenomenon has been analyzed within the framework of Poisson point processes for brittle (Denoual et al., 1997; Grady, 2006) and ductile materials (Trumel et al., 2009). It will be used herein in a unified way to analyze dynamic fragmentation.

The aim of the present paper is to introduce characteristic parameters describing the fragmentation of brittle and ductile materials within the framework of continuum damage mechanics (Lemaitre, 1992). In Section 2 the Poisson–Weibull formalism is introduced to account for the inception and growth of cracks and pores. Governing equations are derived to estimate the activated and growing site densities. Characteristic parameters are then introduced in Section 3 for quasi-static and dynamic loading conditions. Various uses of the characteristic parameters are shown in Section 4.

2. Probabilistic framework

The physical processes of inception and growth are complex under dynamic loading conditions. Wave propagation leads to nonuniform stress fields at the scale of the studied volume element. Due to the material microstructure additional fluctuations arise because of its heterogeneous nature. Inception will be described hereafter when the local stress $\sigma(\mathbf{x}, t)$ exceeds an inception threshold $\sigma_{inc}(\mathbf{x})$. The stress σ corresponds to the maximum principal stress σ_1 for brittle materials, and the hydrostatic stress σ_h for ductile materials.

Once inception has occurred, an initiated crack or a nucleated void starts to grow. The growth process is accompanied by a relaxation zone in which the local stresses decrease (and are no longer equal to the levels that would have been reached had inception not occurred). Outside the relaxation zone the stress field is assumed to be unaltered by the presence of the growing cavity or propagating crack. In the context of continuum damage mechanics (Lemaitre, 1992), the relaxation zones define the damaged regions in which the material no longer sustains the applied stresses.

2.1. Description of inception

To account for such complex situations, a very simple probabilistic model is introduced. All the heterogeneities are lumped into a random inception stress when considering the macroscopic stress field (Trumel et al., 2009). The number N of sites where the applied stress exceeds the inception threshold is assumed to follow a Poisson point process. The probability P of finding $N = \nu$ sites in a uniformly loaded domain Ω reads

$$P(N = \nu, \Omega) = \frac{\Lambda^\nu}{\nu!} \exp(-\Lambda), \tag{1}$$

where Λ is the average number of sites in Ω. The intensity of the Poisson point process is defined by

$$\lambda = \frac{\Lambda}{\mu_n(\Omega)}, \tag{2}$$

where μ_n is the Lebesgue measure in $\mathbb{R}^n$ (*i.e.* length of Ω when $n = 1$, surface of Ω when $n = 2$, and volume of Ω when $n = 3$). The intensity λ corresponds to the density of sites that may initiate a crack or nucleate a cavity. If the domain Ω is not uniformly loaded, the previous expression can be extended to

$$\Lambda = \int_\Omega \lambda(\mathbf{x}) \mathrm{d}\mathbf{x}. \tag{3}$$

In the following, it will be assumed that the intensity λ of the inception process will depend on the applied stress level, namely, the higher the stress level, the higher the intensity as more sites will satisfy the inception condition (*i.e.* $\sigma(\mathbf{x}, t) \geqslant \sigma_{inc}(\mathbf{x})$).

2.2. Inception probability

A first consequence of this type of modeling is related to the additional assumption of weakest link statistics (Pierce, 1926; Freudenthal, 1968). The inception probability corresponds to the first inception event. The probability P_{inc} of finding at least one inception site reads

$$P_{inc} = P(N \geqslant 1, \Omega) = 1 - P(N = 0, \Omega) = 1 - \exp(-\Lambda). \tag{4}$$

This type of hypothesis is known to accurately describe the behavior of brittle materials when subjected to quasi-static loading histories (Weibull, 1939, 1951). If a Weibull model is to be retrieved, it follows that

$$\lambda = \lambda_0 \left(\frac{\langle \sigma \rangle}{\sigma_0} \right)^m, \tag{5}$$

where m is the shape parameter (or Weibull modulus), $\sigma_0 / \lambda_0^{1/m}$ the so-called scale parameter, σ the local equivalent stress, and $\langle \cdot \rangle$ Macauley's brackets.

The mean stress at first inception $\overline{\sigma}_i$ is given by

$$\overline{\sigma}_i = \frac{\sigma_0}{(\lambda_0 \mu_n(\Omega) H_m)^{1/m}} \Gamma\left(\frac{m+1}{m} \right) \tag{6}$$

and the corresponding standard deviation $\overline{\overline{\sigma}}_i$

$$\overline{\overline{\sigma}}_i = \frac{\sigma_0}{(\lambda_0 V_{\mathrm{eff}})^{1/m}} \sqrt{\Gamma\left(\frac{m+2}{m} \right) - \Gamma^2\left(\frac{m+1}{m} \right)}, \tag{7}$$

where Γ is the Euler (gamma) function of the second kind, and H_m the stress heterogeneity factor (Hild et al., 1992)

$$H_m = \frac{1}{\mu_n(\Omega)} \int_\Omega \left(\frac{\sigma}{\sigma_m} \right)^m \mathrm{d}\mathbf{x} \quad \text{with } \sigma_m = \max_\Omega \sigma(\mathbf{x}). \tag{8}$$

The product $\mu_3(\Omega) H_m$ corresponds to the so-called effective volume (Davies, 1973), and $\mu_2(\Omega) H_m$ the effective surface (Gy and Guillemet, 1992; Oakley, 1996).

The Weibull parameters used to model the intensity λ of the Poisson point process are representative of the material microstructure, and more precisely the distribution of inception sites. The Weibull modulus mainly characterizes