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A domain-independent interaction integral for linear elastic fracture analysis of micropolar materials

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ABSTRACT

This paper establishes a domain-independent interaction integral (DII-integral) for linear elastic fracture mechanics of micropolar elastic solids. The DII-integral has three amazing features that make it effective for solving the fracture parameters of complex micropolar materials. The first one is that the DII-integral can decouple the stress intensity factors (SIFs) and couple stress intensity factors (CSIFs) both of which are the key fracture parameters charactering the crack-tip asymptotic singular fields. In details, the DII-integral is derived from the *J*-integral by superimposing an actual field and an auxiliary field. By assigning the fracture parameters in the auxiliary field with different values, the SIFs and CSIFs of different crack opening modes can be obtained separately through the DII-integral. The second important feature is that the DII-integral is domain-independent for material nonhomogeneity and discontinuity. Thanks to this feature, the DII-integral becomes extremely effective for the micropolar materials with arbitrary nonhomogeneous properties or complex interfaces. The third feature is that the DII-integral does not contain any derivatives of material properties, which feature facilitate the practical implementation of the DII-integral on complex micropolar materials. Finally, the DII-integral combined with the extended finite element method (XFEM) is employed to solve four representative crack problems and the results show good validity of the DII-integral for complex micropolar materials.

1. Introduction

A number of composite materials, such as particulate, fiber-reinforced, and granular composites, contain complex interfaces that usually act as a source of defects such as cracks and holes during the manufacturing process or in service. As a result, these composite materials are more prone to fracture failure than homogeneous materials. With the development of multi-scale mechanics, there has been increased interest in the correlation of a material microstructure with the development of its local strain fields, as this correlation is believed to influence the macroscale material response (Gonzalez and Lambros, 2013). When the microstructural effects are important, the micropolar theory is more appropriate than the classical elastic theory to describe microstructural effects (Karlis et al., 2007) because the classical continuum mechanics do not incorporate the effect of the intrinsic length into the models, whereas the micropolar continuum takes that effect into account.

Cosserat and Cosserat (1909) first proposed the Cosserat continua which introduces three rotational degrees-of-freedom in addition to translational degrees-of-freedom. However, the Cosserat continuum theory did

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not receive much attention at that time. Until the 1960s, the Cosserat continua were elaborately studied by Eringen (1966, 1968) who renamed the Cosserat continuum theory with the micropolar continuum theory. Cowin (1969, 1970) introduced a coupling number to character a continuous transition from classical elastic theory to micropolar theory and an internal length which should be able to analytically predict the size effect. The predictions of size effects aroused a great interest to the experimentalists (Gauthier and Jahsman, 1975; Yang and Lakes, 1982; Lakes, 1986, 1995a,b). The experiments showed that the specimens behaved more and more stiffly than expected from the classical elasticity in the tension of the slender cylinders and in the bending of plates and beams, but the size effects were not observed in tension. In particular, the micropolar elastic theory accurately fits the experimental data for the effective stiffness of the bone sample from the osteon to the whole bone (Lakes, 1995b).

Meanwhile, a number of research works on micropolar fracture mechanics were presented. The first investigation was performed by Sternberg and Muki (1967) who considered a crack in an infinite two-dimensional (2D) linear couple stress medium, i.e., a constrained micropolar medium. They derived the crack-tip asymptotic expressions of the stress and couple stress fields, both of which behave an inverse square root singularity. Atkinson and Leppington (1974, 1977) analyzed the effect of couple stresses on the stress concentration and the energy release rate of a crack in couple stress and micropolar elasticities. They showed that the crack-tip stress and couple stress fields also exhibit an inverse square root singularity in micropolar media. The stress intensity factors (SIFs) and couple stress intensity factors (CSIFs) were investigated by applying Hankel transforms and dual integral equations for penny-shaped and Griffith cracks in an infinite micropolar elasticity (Paul and Sridharan, 1980, 1981) and for an insulated penny-shaped crack in an infinite micropolar thermo-elasticity under uniform heat-flow loading (Sridharan, 1983). They found that the crack-tip stress environment depends on both the material intrinsic length and the coupling number, both of which characterize the influences of microstructures. Yadava et al. (1994) analyzed a penny-shaped interface crack between two bonded dissimilar micropolar elastic half-spaces subjected to internal pressure. They showed that similarly to a crack located along the interface of two classical elastic half-spaces, the crack-tip stress field exhibits an oscillating singularity. Jaric (1990) established a path-independent *J*-integral for a nonlocal micropolar continuum and showed that the *J*-integral has a physical meaning of the energy release rate. The conservation integrals for a crack in a micropolar elasticity were rigorously derived from Noether's theorem by Lubarda and Markenscoff (2000, 2003). Diegele et al. (2004) derived the asymptotic expressions of the crack-tip fields for mixed-mode cracks in an isotropic micropolar elastic solid, and analyzed the effects of material parameters on the SIFs and CSIFs. They showed that two SIFs (modes I and II SIFs) are involved in the in-plane crack-tip stress field, while only one CSIF (mode I CSIF) is evolved in the in-plane crack-tip couple stress field. In numerical aspects, Kennedy (1999) employed the finite element method (FEM) to predict failure in particulate or fiber-reinforced composite structures containing a notch which was taken to be a circular hole, an elliptical hole and a sharp crack, sequentially. Dillard et al. (2006) established an anisotropic compressible plasticity model incorporating intrinsic length scale effects and applied the model to simulate the crack propagation in a nickel foam plate. Except the FEM, the boundary element method (BEM) (Shmoylova et al., 2007) and the extended finite element method (XFEM) (Khoei and Karimi, 2008; Kapiturova, 2013) were developed to analyze different crack problems of micropolar materials. Recently, many interesting analytical and numerical investigations were carried out on the fracture problems of micropolar materials and structures (Yavari et al., 2002; Midya et al., 2007; Warren and Byskov, 2008; Li and Lee, 2009; Piccolroaz et al., 2012; Korepanov et al., 2012; Goda et al., 2012).

Although the elastic crack-tip fields and the related conservation integrals have been described clearly in many published articles, to the best knowledge of the authors, no research work shows how to decouple the SIFs and CSIFs of different modes from the conservation integrals, which brings a certain regret to linear elastic micropolar fracture mechanics. In the previous studies (Yu et al., 2009, 2010, 2012, 2014), the authors have established a domain-independent interaction integral (DII-integral) for classical elastic, piezoelectric, and magneto-electro-elastic materials to decouple linear elastic fracture parameters. The DII-integral has two advantages: first, it is domain-independent for interfaces; secondly, it does not contain any derivative of material properties. These advantages can greatly facilitate the practical implementation of the DII-integral on the materials with complex interfaces. Therefore, this paper aims to develop a DII-integral to decouple the mode I SIF, mode II SIF and mode I CSIF.

The outline of this paper is as follows. Section 2 briefly reviews the micropolar theory, introduces the definition of the interaction integral, and shows how to extract the SIFs and CSIFs from the interaction integral. Section 3 derives the domain form of the interaction integral and then, establishes the DII-integral. Section 4 briefly describes the XFEM for micropolar materials and the numerical discretization of the DII-integral. In Section 5, four representative cracked micropolar plates are investigated to show the validity of the DII-integral. Finally, a summary is given in Section 6.

2. Fracture mechanics of micropolar elasticity

In classical continuum mechanics, at each point only three translational degrees-of-freedom are described while three microrotations ϕ_i are considered to be independent of displacement components u_i in the micropolar theory.

2.1. Basic equations

The basic equations of a centrosymmetric isotropic micropolar elasticity are as follows (Lakes and Benedict, 1982; Yavari et al., 2002; Diegele et al., 2004):