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A micro–macro approach to rubber-like materials. Part II: The micro-sphere model of finite rubber viscoelasticity

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Abstract

A micromechanically based non-affine network model for finite rubber elasticity incorporating topological constraints was discussed in Part I (2004. J. Mech. Phys. Solids 52, 2617–2660) of this work. In this follow-up contribution we extend the non-affine micro-sphere model towards the description of time-dependent viscoelastic effects. The viscoelastic network model is constructed by an additive split of the overall response into elastic equilibrium-stress and viscoelastic overstress contributions. The equilibrium response of the network is understood to be related to results obtained from an infinite relaxation process and modeled by our above mentioned elasticity formulation. Inspired by (2004. J. Mech. Phys. Solids 52, 2617–2660), the rate-dependent overstress response is assumed to be driven by *two micro-kinematical mechanisms* related to the stretch and the area contraction of a tube containing a prototype chain. Firstly, a *retraction of fictitiously unconstrained dangling chains* is explained by diffusive reptile motions. Secondly, a *release of constraint effects* due to surrounding chains is modeled by a time-dependence of a tube cross-section area. The latter contribution is considered to be a result of the retraction of forest chains. We outline a distinct micromechanical model for the viscous overstress in terms of the above outlined two micro-kinematic mechanisms and discuss its numerical implementation in context of an *affine homogenization procedure* of space orientations. The characteristics and modeling capabilities of the proposed micro-sphere model of finite rubber viscoelasticity are reported for a broad

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spectrum of experimentally-based benchmark simulations. They demonstrate an excellent performance of the model in simulating rate and hystereses effects of rubbery polymers.

Keywords: Micromechanics; Rubber viscoelasticity; Network models; Experiments; Simulation

1. Introduction

Recent developments in the production techniques of polymers have lead to a broad use of polymer products in industrial applications, for example medical rubber products, tire technology, automotive industry. Advanced production methods, however, have brought along a highly nonlinear mechanical behavior of these materials with respect to different loading programs at various rates. In context of the phenomenological material modeling on a macroscopic continuum level, dissipative inelastic phenomena observed in polymers are generally referred to finite-strain elasto-visco-plasticity including deformation induced damage effects. The challenges encountered in the modeling of these phenomena have been tackled by many researchers from different disciplines, in the engineering community in a more macroscopic sense and by scientists from physical chemistry and chemical physics at the molecular level. However, there have been also several attempts to combine both approaches based on a multiscale modeling. In the context of rubber elasticity, such an attempt is the non-affine network model recently proposed by Miehe et al. (2004). It combines elementary constitutive assumptions on the chain-level with directly evaluated micro-to-macro-scale transitions towards the description of the overall network response. As shown in Miehe et al. (2004), the model provides an excellent fit of available experiments. In this second part of our work on *micro-sphere models for rubbers*, we extend this formulation towards time-dependent viscous effects by constructing an internal variable formulation in terms of microscopic overstress fields defined on the micro-sphere.

1.1. Basic experiments and classification of the material response

The overall behavior of rubbery polymer aggregates exhibits highly non-linear finite elasticity combined with inelastic effects. Fig. 1 shows the response of a cylindrical hyrogenated nitrile butadiene rubber HNBR50 specimen, produced by the Robert Bosch GmbH, under cyclic uni-axial loading at room temperature. In order to exclude the Mullins-type softening effect due to a partial breakdown of the microstructure, a *pre-conditioning* (deformation-induced pre-damaging) was performed as described for example in James and Green (1975); Lion (1996); Bergström and Boyce (1998) and Miehe and Keck (2000). Following Haupt (1993), the hysteresis in Fig. 1 can be explained either elasto-visco-plasticity (response with equilibrium hysteresis) or by viscoelasticity (response without equilibrium hysteresis). In order to investigate the existence of a possible equilibrium hysteresis,