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Investigation of frictional sliding contact problems of triangular and cylindrical punches on monoclinic piezoelectric materials

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ABSTRACT

A theoretical model of a frictional sliding contact problem for monoclinic piezoelectric materials under triangular and cylindrical punches is established. The characteristic equation related to the governing equations of monoclinic piezoelectric materials is of eight-order, which generates real and/or complex eigenvalues. Fundamental solutions that can lead to real values of physical quantities are given for both real and complex eigenvalues. By applying Fourier transform, the mixed boundary value problem is reduced to a singular integral equation of the second kind of Cauchy type. Based on exact solutions of the reduced singular integral equation, closed-form expressions of various surface stresses and electric displacement are obtained. Moreover, relations between the applied load and the contact region are obtained. Numerical results are given to show the influences of the friction coefficient on various surface stresses, electric displacement and even the width of the contact region. The underlying physics/mechanics accounting for the observations are presented.

1. Introduction

Being smart materials, piezoelectric materials are initially used as the constituents of sensors and actuators in advanced control system due to their electro-mechanical coupling effects. With the rapidly increasing importance, piezoelectric materials are also applied to many other territories such as medical engineering, aeronautical and astronautical industries, and so on. An electro-mechanical analysis is vital for the design of piezoelectric materials and has absorbed the attention of many researchers. Basic solutions involving transversely isotropic piezoelectric materials were obtained (e.g. Gong and Suo, 1996; Guo et al., 2006; Li and Lee, 2010; Narita et al., 2004; Shindo et al., 1998; Shindo et al., 2004; Wang, 2004; Wang and Han, 2007).

Because piezoelectric materials are naturally anisotropic and in-plane and anti-plane deformations often couple each other, it is necessary to perform the electro-mechanical analysis involving anisotropic piezoelectric materials. Du et al. (1994) obtained the electro-mechanical coupling fields of a 2-D anisotropic piezoelectric medium with an elliptic inclusion by extending the Stroh's formalism for anisotropic linear elasticity to piezoelectricity. An exact electro-elastic analysis for general anisotropic piezoelectric media subjected to a line force and a line charge was performed (Liu et al., 1997) by using the plane wave decomposition method and a subsequent application of the residue calculus. Stress concentrations due to defects can give rise to critical crack growth and subsequent mechanical failure when piezoelectric materials are under

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Nomenclature

$-a$, b	edges of the punch
$c_{\alpha\beta}$	elastic constants
C_T	slope of the triangular punch
D_0	factor used to normalize the electric displacement
D_m	electric displacement components
$e_{\alpha\beta}$	piezoelectric constants
E_α	electric field components
H	determinant of matrix M_H
H_{kj}	complementary minors of matrix M_H
i	imaginary unit
Im[ ]	imaginary part
M_j	unknown functions to be determined
M_H	matrix obtained after using boundary conditions
N_1	number of positive real roots
N_2	number of pairs of complex conjugate roots with positive real part
$p(x)$	surface contact stress
P	normal loading acting on the punch
$P_j^{(\tau,\vartheta)}(t)$	Jacobi Polynomials
$q(x)$	surface shear stress
Q	shear loading acting on the punch
R	radius of the cylindrical punch
$Re[\ ]$	real part
u, v, w	displacements
U, V, W	displacements in the Fourier transformed domain
$w_0(x)$	penetration depth
x, y, z	spatial coordinates
$\epsilon_{\alpha\alpha}$	dielectric constants

Greek symbols

$\varepsilon_{\alpha\beta}$	strain components
ϕ	electric potential
Φ	electric potential in the Fourier transformed domain
η, η_k	eigenvalues
κ_0	index of the singular integral equation
Λ_{mn}	asymptotic terms
μ^*	factor used to normalize stresses under a triangular punch
μ_f	friction coefficient
$\varpi(t)$	weight function of Jacobi Polynomials
ϑ	power of stress singularity at the leading edge
σ_0	factor used to normalize stresses under a cylindrical punch
σ_{mn}	stress components
τ	power of stress singularity at the trailing edge
ω	Fourier transform variable

the action of mechanical and electrical loadings. A generalized plane problem of a finite Griffith crack moving in a general anisotropic piezoelectric material was addressed in Ref. Soh et al. (2002). Electric fields may be generated due to static or moving dislocations in piezoelectric media. Exact solution for an infinite straight dislocation in a steady state in anisotropic piezoelectric materials was obtained (Faivre and Saada, 1972). Using Stroh's formalism, Barnett and Lothe (1975) obtained the electroelastic fields produced by a static dislocation in an anisotropic piezoelectric insulator. The fields of stress and displacement produced by dislocation in a three dimensional anisotropic piezoelectric medium were also computed (Minagawa, 2004). Closed-form expressions of electroelastic fields generated by a moving screw dislocation in hexagonal piezoelectric solids were derived (Wang and Zhong, 2002). An analysis for the generalized plane problem of anisotropic piezoelectric materials with general types of dislocation moving at a constant subsonic velocity was performed to reveal the basic properties of the dislocation-induced piezoelectric polarization and electric fields (Soh et al., 2005).

Besides the crack and dislocation problems mentioned above, contact problem is of significant importance in the designing of piezoelectric materials. In many practical applications involving piezoelectric materials, such as applications in aerospace, mechanical, electrical, civil and biomedical engineering (Smith, 2005), contact phenomena can occur or may be used on purpose, e.g. for measurement devices. Few studies on the contact behavior of anisotropic piezoelectric materials have been done due to the mathematical difficulties encountered. The contact problem of monoclinic piezoelectric materials exhibiting hexagonal symmetry, a kind of anisotropic piezoelectric materials, was investigated by Ramirez and Heyliger (2003) by employing the local/global stiffness matrix approach. By using the same approach, Ramirez (2006) investigated the contact problem for monoclinic piezoelectric materials which can generate complex eigenvalues. In papers written by Ramirez and Heyliger (2003) and Ramirez (2006), the contact was smooth and numerical solutions were given. Exact solutions of the frictional contact problem of anisotropic piezoelectric materials could greatly benefit the designing and applications of piezoelectric materials.

Motivated by the above-mentioned reasons, the present paper performs an exact analysis of the frictional sliding contact problem for monoclinic piezoelectric materials under the action of a rigid punch, which occupies either a triangular profile or a cylindrical profile. Coulomb frictional law is applied inside the contact region. Fundamental solutions that can lead to real values of physical quantities are obtained for both real and complex eigenvalues. The stated problem is reduced to a singular integral equation of the second kind of Cauchy type, which is solved exactly. Explicit expressions of various surface stresses and electric displacement are readily given based on the exact solution. Moreover, relationships between the applied load and the contact region are obtained. Numerical results are given to show the distribution of various surface stresses, electric displacement and width of the contact region with the val-