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A new constitutive theory for fiber-reinforced incompressible nonlinearly elastic solids

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Abstract

We consider an incompressible nonlinearly elastic material in which a matrix is reinforced by strong fibers, for example fibers of nylon or carbon aligned in one family of curves in a rubber matrix. Rather than adopting the constraint of fiber inextensibility as has been previously assumed in the literature, here we develop a theory of fiber-reinforced materials based on the less restrictive idea of *limiting fiber extensibility*. The motivation for such an approach is provided by recent research on *limiting chain extensibility* models for rubber. Thus the basic idea of the present paper is simple: we adapt the limiting *chain* extensibility concept to limiting *fiber* extensibility so that the usual inextensibility constraint traditionally used is replaced by a *unilateral constraint*. We use a strain-energy density composed with two terms, the first being associated with the isotropic matrix or base material and the second reflecting the transversely isotropic character of the material due to the uniaxial reinforcement introduced by the fibers. We consider a base neo-Hookean model plus a special term that takes into account the limiting extensibility in the fiber direction. Thus our model introduces *an additional parameter*, namely that associated with limiting extensibility in the fiber direction, over previously investigated models. The aim of this paper is to investigate the mathematical and mechanical feasibility of this new model and to examine the role played by the extensibility parameter. We examine the response of the proposed models in some basic

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homogeneous deformations and compare this response to those of standard models for fiber reinforced rubber materials. The role of the strain-stiffening of the fibers in the new models is examined. The enhanced stability of the new models is then illustrated by investigation of cavitation instabilities. One of the motivations for the work is to apply the model to the biomechanics of soft tissues and the potential merits of the proposed models for this purpose are briefly discussed.
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Keywords: Fiber-reinforced incompressible nonlinearly elastic solids; Transversely isotropic material; Constitutive models; Limiting fiber extensibility

1. Introduction

We consider a nonlinearly elastic material in which a matrix is reinforced by strong fibers, for example fibers of nylon or carbon aligned in one family of curves in a rubber matrix. In the literature, a highly idealized mathematical model for such a material has been developed on considering the internal kinematical constraints of fiber *inextensibility* and incompressibility. The fibers are treated as line elements not occupying any space. These assumptions lead to a simple theory which permits the analytical solution of many boundary-value problems by exact or approximate methods and, moreover, by exaggerating the properties of strongly anisotropic materials this theory highlights some of the special behavior of these solids (Spencer, 1972).

On the other hand, it is clear that there are many applications where the idealization of an inextensible material is too restrictive. For this reason it is of interest to develop a theory of fiber-reinforced materials based on the less restrictive idea of *limiting fiber extensibility*. The motivation for such an approach is provided by recent research on *limiting chain extensibility* models. In the *molecular theory* of elasticity (see, e.g. Erman and Mark, 1997; Erman, 2004) macroscopic models (i.e. functional forms of the strain-energy density W) are obtained from microscopic considerations. Starting from the elastic free energy of a single chain, it is possible to compute the total elastic free energy of the network composing the mesoscopic structure of the elastomer by a suitable averaging procedure of all the elastic free energies of the individual chains. To derive the elasticity of a single chain it is necessary to introduce a distribution function for the end-to-end distance $\mathbf{r}$ of the polymeric chain. To take into account the finite chain extensibility of the polymeric chains this distribution function must be non-Gaussian, because the Gaussian distribution assigns a nonzero probability to any length. The most widely used and simplest non-Gaussian probability distribution $P(\mathbf{r})$ with compact support (i.e. $P(\mathbf{r}) = 0$ if $\mathbf{r} > \mathbf{r}_{max}$) is due to Kuhn and Grün (1942). To consider the idea of limiting chain extensibility from a *phenomenological* point of view there are several possibilities (see, e.g. Horgan and Saccomandi, 2002a). One of the *simplest* phenomenological constitutive models for incompressible materials with limiting chain extensibility is due to Gent (1996, 1999) who proposed the

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