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Nonlinear analyses of wrinkles in a film bonded to a compliant substrate

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Abstract

Subject to a compressive membrane force, a film bonded to a compliant substrate often forms a pattern of wrinkles. This paper studies such wrinkles in a layered structure used in several recent experiments. The structure comprises a stiff film bonded to a compliant substrate, which in turn is bonded to a rigid support. Two types of analyses are performed. First, for sinusoidal wrinkles, by minimizing energy, we obtain the wavelength and the amplitude of the wrinkles for substrates of various moduli and thicknesses. Second, we develop a method to simultaneously evolve the two-dimensional pattern in the film and the three-dimensional elastic field in the substrate. The simulations show that the wrinkles can evolve into stripes, labyrinths, or herringbones, depending on the anisotropy of the membrane forces. Statistical averages of the amplitude and wavelength of wrinkles of various patterns correlate well with the analytical solution of the sinusoidal wrinkles.

Keywords: Wrinkles; Thin film; Elastomer; Nonlinear plate; Elasticity

1. Introduction

Consider a film of a stiff material (e.g., a metal) on a substrate of a compliant material (e.g., an elastomer). Subject to a compressive membrane force, the film may

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form a pattern of wrinkles, and remain bonded to the substrate (Bowden et al., 1998). The wrinkles are a nuisance in some applications (Iacopi et al., 2003), but may be used as stretchable interconnects (Watanabe et al., 2002; Lacour et al., 2004), as templates for device fabrication (Yoo et al., 2002; Harrison et al., 2004), or as a means to evaluate mechanical properties of materials (Stafford et al., 2004).

For a film partially debonded from a substrate, when the compressive membrane force exceeds a critical value, the film buckles like an edge-clamped plate; the critical membrane force depends on the size of the debond (Hutchinson and Suo, 1991). As the magnitude of the initial membrane force increases, the debonded film buckles into a complex pattern, and the wavelength decreases (Gioia and Ortiz, 1997; Audoly, 2000). The buckled film may behave like a sheet of crumpled paper (Lobkovsky et al., 1995; Ben Amar and Pomeau, 1997).

By contrast, when a wrinkled film remains bonded to a compliant substrate, the critical membrane force depends on the modulus of the substrate (Allen, 1969). Above the critical condition, as observed experimentally, the bonded film wrinkles into patterns such as stripes, labyrinths and herringbones (Fig. 1), but the wavelength of the individual wrinkles remains practically unchanged as the amplitude of the wrinkles increases (Ohzono and Shimomura, 2004).

This paper focuses on wrinkles in a substrate-bonded film. Specifically, the film is bonded to a compliant substrate, which in turn is bonded to a rigid support (Fig. 2). The substrate thickness, H, can vary by orders of magnitude in applications. The

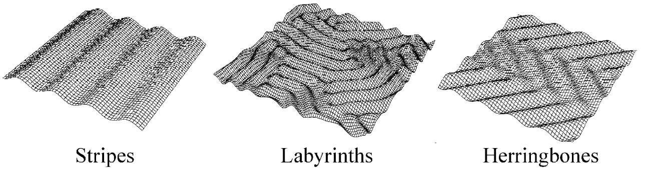

Fig. 1. Schematics of three representative patterns of wrinkles: stripes (a periodic array of straight wrinkles), labyrinths (disordered zigzag wrinkles), and herringbones (a periodic array of zigzag wrinkles).

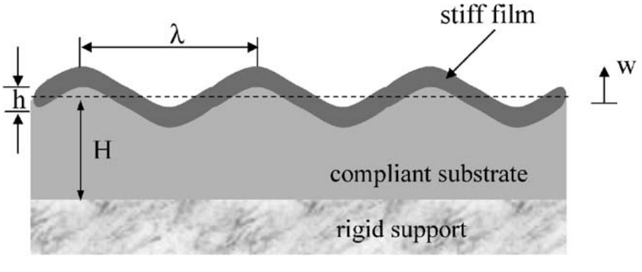

Fig. 2. A stiff film is bonded to a compliant substrate, which in turn is bonded to a rigid support. The film is under in-plane compressive membrane forces. The wrinkle wavelength λ is much larger than the film thickness h. However, the ratio of the wavelength to the substrate thickness, λ/H, can vary from a small fraction to a large number.