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On the fracture behaviour of an interface crack with plastic zone corrections

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A B S T R A C T

In this paper, the mixed-mode Dugdale model is extended to investigate an interface crack problem. The plastic zone size and the crack tip opening displacement for an interface crack between two dissimilar materials under remote uniform tensile loading are examined. In the solution procedure, continuously distributed edge dislocations are applied to simulate the crack, and the physical problem is formulated into the singular integral-equation of 2nd kind. In the numerical examples, the effects of the normalized uniform tensile loads and the Dundurs' parameters β on the normalized plastic zone size and the normalized crack tip opening displacement are analyzed in detail. The obtained results show that the normalized plastic zone size and the normalized crack tip opening displacement are symmetrical about the axis $\beta = 0$. Further, the relationships among the plastic zone size, the crack tip opening displacement and Dundurs' parameters are curve-fitted as two polynomial functions which are highly user-friend for engineers in solving practical interface crack problems.

1. Introduction

Layered materials (structures) are widely applied in structural engineering and microelectronic engineering. During the manufacturing process of these layered materials, small cracks may be introduced at the interface between dissimilar materials because of the mismatches of thermal and mechanical properties (shown in Fig. 1). Obviously, these interface cracks have a great deal of influence on the mechanical functioning of layered materials (structures) as they tend to cause stress and strain concentrations which may lead to failure of the materials (or structures).

Theoretical investigations into elastic interface crack problems began in late 1950s. The nature of the stress and displacement fields near the tip of an interface crack between two dissimilar semi-infinite elastic media was evaluated by Williams (1959), Erdogan (1963) and England (1965). Rice (1988) defined the complex stress intensity factor. The work above was based on the open model (traction-free crack surfaces) which resulted in the stress oscillation ahead of the crack tip and the physically impossible crack surface interpenetration. To overcome the difficulty, the contact model was proposed and developed by Comninou (1977), Atkinson (1982) and Gautesen and Dundurs (1987, 1988). In this model, contact zones were assumed behind the crack tips where the crack surfaces were assumed to be compressed together, but were not permitted to interpenetrate. Based on the assumption that there are zones of bounded cohesive tensile and shear stresses near a crack tip, Achenbach et al. (1979) constructed a model to analyze adhesive bond failure at the tip of an interface crack. Elastic and plastic analysis of interface cracks began in the late 1980s. Shih and Asaro (1988) studied the complex nonlinear stress–strain fields near the tip of a crack at the interface between an elastic–plastic medium and a rigid substrate by the finite element method. With

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the contact model, Aravas and Sharma (1991) presented elastic and plastic solutions for a plane strain crack at the interface between a homogeneous isotropic elastic–plastic material and a rigid substrate. Sladek and Sladek (1995) presented the formulation of the boundary element method for solving interface crack problems. In their study, the plastic zone size, the path independence J- and L-integrals of a straight interface crack and a circular arc-shaped interface crack under uniform tension were investigated. Sham et al. (1999) derived asymptotic expressions for the stress fields near the tip of a plane strain crack at the interface between two incompressible elastic-perfectly plastic solids with matching elastic properties but mismatched yield strengths. Lee and Kim (2001) investigated the effects of the T-stress and plastic mismatch on the interface crack tip stress fields by the finite element method. Belhouari et al. (2008) investigated the interaction of an interfacial main crack and a sub-interfacial microcrack in ceramic/metal bi-materials.

However, the complicated elastic–plastic stress analysis mentioned above may be difficult to apply in engineering practice directly. Meanwhile, fracture analyses based on the theory of elasticity may be more applicable to brittle materials. For ductile materials (such as metal-matrix composites), fracture analyses will be more accurate if plastic zone corrections at crack tips are made, and the crack tip opening displacement criterion can be better used to judge if a fracture will take place. Particularly for interface crack problems, it is not practical for engineers to use complex stress intensity factors as fracture parameters. Based on this logic, in our current study, the mixed-mode Dugdale model (Becker and Gross, 1988) is applied to analyze the physical problem (as shown in Fig. 2) of an interface crack. Recently, the Dugdale model and the mixed mode Dugdale model were applied to calculate the plastic zone size and the crack tip opening displacement for a crack near a circle inclusion or paralleling an interface between dissimilar materials (Hoh et al., 2012; Yi et al., 2011).

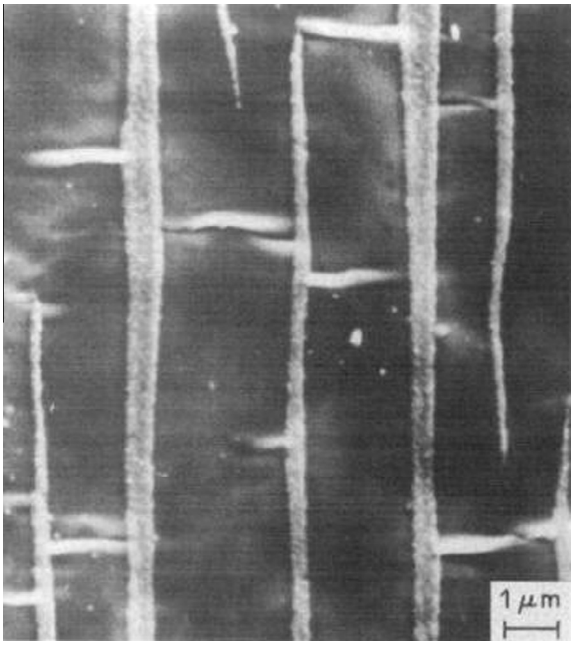

Fig. 1. Micrograph of cracked silica films on copper substrates strained at 500 °C (Jobin et al., 1992). The parallel thick lines are the opening parallel cracks. The dark lamellae are the silica strips still attached to the copper substrate.

The objective of the current research work is to obtain the plastic zone size and the crack tip opening displacement for an interface crack under remote uniform tensile loading. Two user-friendly polynomial functions are obtained from curve-fitting to calculate the plastic zone size and the crack tip opening displacement.

2. The current model with plastic zone corrections

The mixed-mode Dugdale model is applied to analyze the current physical problem (shown in Fig. 2). A long, slim plastic zone is assumed at each crack tip. The crack length is taken as $2L$, ρ is the plastic zone length. The plane stress condition is considered. The stresses applied in the plastic zones include the normal stress σ_y and the shear stress τ_{xy} and they satisfy the Von Mises yield criterion having the form as

$$\sqrt{\sigma_y^2 + 3\tau_{xy}^2} = \sigma_{ys} \tag{1}$$

where σ_{ys} is the lower yielding stress of the two solids. The length of plastic zone ρ, the normal stress σ_y and the shear stress τ_{xy} can be determined when the stress singularity vanishes (shown in Fig. 2):

$$K + K_\rho = 0 \tag{2}$$

Here, K is the stress intensity factor caused by the applied load σ_0, K_ρ is the stress intensity factor caused by the normal stress σ_y and the shear stress τ_{xy} in the plastic zones. For example, at the right crack tip, if the applied load σ_0 and the lower yielding stress of the two solids σ_{ys} are given, the length of the plastic zone ρ, the normal stress σ_y and the shear stress τ_{xy} in the right plastic zone can been determined by meeting Eqs. (1), (2) (in fact, Eq. (2) is equivalent to two equations considering its real and imaginary parts). Specially, when the crack is embedded in an infinite homogenous material under a remote uniform tensile load, the shear stresses in the plastic zones are zero and the current model will reduce to the conventional Dugdale model.

The dislocation expression for the model is

$$\begin{array}{ll} -\beta B(x) - \frac{i}{\pi}\int_{-L-\rho}^{L+\rho}\frac{B(\xi)}{x-\xi}d\xi = \frac{\tilde{\sigma}_{yy}(x) - i\tilde{\sigma}_{xy}(x)}{C} & -L-\rho < x < L+\rho \\ \tilde{\sigma}_{yy}(x) = -\sigma_0, \tilde{\sigma}_{xy}(x) = 0 & -L < x < L \\ \tilde{\sigma}_{yy}(x) = \sigma_y - \sigma_0, \tilde{\sigma}_{xy}(x) = -\tau_{xy} & -L-\rho < x < -L \\ \tilde{\sigma}_{yy}(x) = \sigma_y - \sigma_0, \tilde{\sigma}_{xy}(x) = \tau_{xy} & L < x < L+\rho \end{array} \tag{3}$$

Here, $B(=B_x + iB_y)$ is dislocation density, β is one of the Dundurs' parameters, given by

$$\beta = \frac{\mu_2(\kappa_1 - 1) - \mu_1(\kappa_2 - 1)}{\mu_2(\kappa_1 + 1) + \mu_1(\kappa_2 + 1)} \tag{4}$$