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Grid coarsening, simulation of transport processes in, and scale-up of heterogeneous media: Application of multiresolution wavelet transformations

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Abstract

We describe a recently-developed approach to the numerical simulation of transport processes in heterogeneous materials and fluid flow in porous media. The morphology of such materials and media is characterized by broadly-distributed and correlated local conductances or permeabilities, hence necessitating its representation by a highly-resolved computational grid that may contain millions of grid points or blocks. Simulation of flow and transport in such grids is very difficult and time consuming. The new approach is based on the use of wavelet transformations. It computes the wavelet scaling and detail coefficients of the conductance or permeability field in the highly-resolved grid. It then utilizes the coefficients to systematically coarsen the grid's low-conductance or permeability blocks while retaining its high resolution in the high-permeability or conductance zones, and those zones in which the quantities of interest, such as the temperature or pressure, experience large variations. The coarsening can be static, generating a fixed grid, but may also be dynamic such that the grid structure evolves with the time if, for example, the local conductances are a strong function of the temperature. The method is applied to study transient flow and transport in heterogeneous materials and porous media, and is shown to lead to accurate solution of the problem at greatly reduced (by at least two orders of magnitude) computational cost.

Keywords: Heterogeneous materials; Porous media; Flow and transport processes; Wavelet transformations; Grid coarsening

1. Introduction

Transport processes in heterogeneous materials constitute an important set of phenomena that, due to their relevance to a wide variety of problems in natural and industrial processes, have been studied for decades. Examples include flow in porous media (for reviews see, for example, Sahimi, 1993, 2005), conduction in, and mechanical properties of, heterogeneous materials (see the extensive discussions by Torquato, 2002; Sahimi, 2003), fracture of disordered solids, and many more. Due to the complexity of their microstructure, the exact solutions of flow and transport equations in disordered

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materials are not available. Although many approximate analytical techniques have been developed (Torquato, 2002; Sahimi, 2003) in order to estimate the effective flow and transport properties, their accuracy, especially for highly heterogeneous materials, is limited. Hence, developing efficient numerical methods for solving the flow and transport equations in a heterogeneous material has been a subject of considerable research for decades.

We consider two important classes of problems in heterogeneous media. One is conduction of heat or electrical charges in a heterogeneous material, characterized by a broad and correlated distribution of the local conductances, or slow flow of a fluid in a *laboratory-scale* disordered porous material. Aside from the analytical approximations (Torquato, 2002; Sahimi, 2003), the effective conductivity g_e or permeability k_e of the material has traditionally been estimated using one of the following three approaches (the same approaches have been used for estimating the effective elastic moduli of disordered materials; see, for example, Torquato, 2002; Sahimi, 2003):

(i) Rigorous and relatively sharp upper and lower bounds to g_e or k_e have been derived (Torquato, 2002; Sahimi, 2003). However, if the local conductances or permeabilities are broadly distributed, such bounds are often not very accurate.

(ii) The conduction or flow equation is discretized by a finite-difference (FD) or finite-element (FE) method, and the resulting set of the discrete equations are solved by an efficient numerical technique. This approach yields accurate solution of the problem. However, with the advent of sophisticated instrumentation, it is now possible to obtain detailed data on the morphology of many heterogeneous materials and laboratory-scale porous media (see, for example, Pinnavaia and Thorpe, 1995), and even develop their detailed three-dimensional (3D) image using X-ray computed tomography (Knackstedt et al., 2001). Utilizing the detailed data in any simulation of flow and transport in disordered materials and porous media would entail using a highly-resolved FD or FE grid containing millions of grid nodes or blocks, hence resulting in millions of discrete equations to be solved, a daunting task. Moreover, if one is to solve a transient flow or transport problem in such materials, then a very large set of equations must be solved repeatedly over a long period of time, an even more difficult task.

(iii) The third approach is based on lattice or network models of a disordered material, popularized by Kirkpatrick (1973), which represents the material's morphology by a lattice or network of bonds, each of which represents a portion of the material at some appropriate length scale. The details of the conduction or flow properties of any portion of the material is then subsumed into a *local* constitutive law, e.g., the Fourier's law of heat conduction or Ohm's law of electrical currents. Disorder is introduced into the model by allowing the local conductances or permeabilities to vary from bond to bond according to a given statistical distribution function that accurately represents the material's morphology. A network of such bonds is then used to numerically calculate the temperature, voltage, or pressure distribution in the material under the application of a macroscopic boundary condition. Similar to the FD or FE methods, simulating transport in a highly heterogeneous material would entail using very large networks.

Consider now a second class of problems, namely, modeling of fluid flow and transport processes in *field-scale* porous media (FSPM), such as oil reservoirs and groundwater aquifers. Such porous media are characterized by large-scale (orders of magnitude) variations in their local permeabilities and the existence of long-range correlations between them. Modern geostatistical techniques (Jensen et al., 2000) enable one to develop realistic and highly-resolved models, in the form of 3D computational grids, to represent FSPM in which the permeability and porosity are broadly distributed, and the stratified structure of the field is accurately represented. The computational grid is usually referred to as the *geological model* of FSPM. However, taking proper account of the correlations and the relevant length scales would entail using a computational grid that contains millions of grid points or blocks, hence resulting in millions of discrete (and highly-nonlinear) equations that describe the fluid flow and transport phenomena in the system, to be solved over thousands of time steps, a very difficult, if not impossible, task.

It is, therefore, clear that detailed simulation of both classes of problems requires using very large