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A granular material with a negative Poisson's ratio

N. Gaspar

DST/DPD, F161.2/Rm210, AWE, Aldermaston, Reading, RG7 4PR, UK

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ABSTRACT

An auxetic material is one with a negative Poisson's ratio and has the property of widening when stretched or narrowing when compressed in contrast to conventional materials. Typical auxetic materials include modified polymeric foams (Lakes, 1987), regular honeycombs, polypropylene fibres and certain crystal structures. Auxetic materials benefit from enhanced mechanical properties. These benefits include improved indentation resistance, enhanced shear moduli and fracture toughness Evans and Alderson (2000).

While modelling granular materials, Bathurst and Rothenburg (1988a,b) noted the theoretical possibility of a negative Poisson's ratio if the constitutive grains had unconventional interactive properties. These unconventional interactive properties are that the tangential interaction should be stiffer than the normal interaction. In this work, a 2D unit cell has been designed with just such an unconventional interaction so that a 2D granular material can be constructed with negative Poisson's ratio. Theoretical calculations of such a granular assembly are made using mean strain assumptions and first order heterogeneity calculations. These are compared to 2D discrete element simulations, finite element simulations and Bathurst and Rothenburg's original result.

1. Introduction

Auxetic foams were first manufactured in 1987 and quickly became of interest to the science community. The interest was due to the fact that it was the first example of a manufactured material with a negative Poisson's ratio: a material that expands laterally when stretch longitudinally. A negative Poisson's ratio imparts to a material many proven and postulated benefits: enhanced fracture toughness (Lakes, 1987), indentation resistance (Alderson and Evans, 1994), acoustic response (Lipsett and Beltzer, 1988), dynamic tuning of electro-magnetic permittivity (Smith et al., 2000) and variable porosity (Rasburn et al., 2001).

Since then much work has established a broad spectrum of auxetic materials, yet little progress has been made in starting large scale use and manufacture. This is partly because of the difficulty in making reliable and predictable auxetic materials. Notably the novel properties of many auxetic materials, in particular polymeric foams, comes from a complicated, non-trivial, 3D structure within the fabric of the material (microstructure). Understanding this microstructure, how it behaves under use and how it is formed in the manufacture are key to auxetic materials moving from research laboratories to industrial and commercial environments.

There are broadly three mechanisms for generating a negative Poisson's ratio.

(i) A microstructure that contains unfolding (beam or plate) elements. Lakes (1991) first suggested this as a regular array such as the honeycombs in Fig. 1. You can also have a disordered array as shown in the micrograph of an auxetic foam in Fig. 2.
(ii) A microstructure of contacting elements with the special condition that the tangential interaction should be stiffer than the normal interaction; the Bathurst and Rothenburg result.

E-mail address: neil@ngaspar.info

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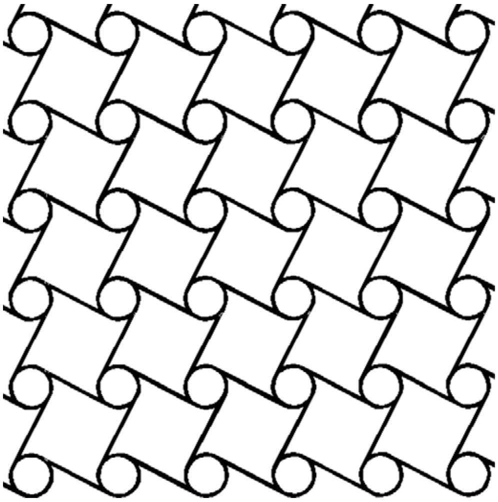

Fig. 1. A chiral honeycomb that exhibits a negative Poisson's ratio.

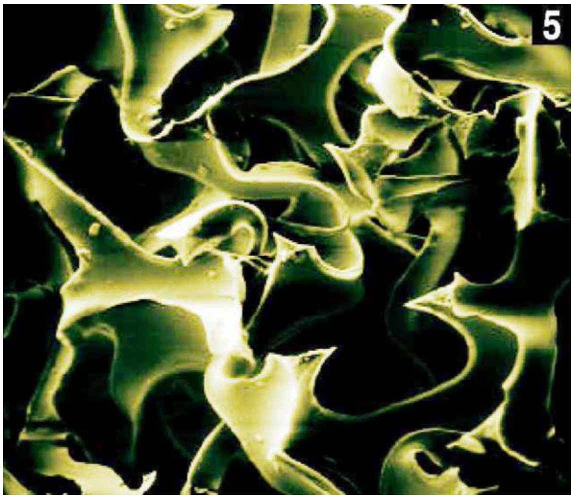

Fig. 2. Micrograph of an auxetic foam (Smith et al., 2000).

(iii) A heterogeneous media with a wide spread of elastic properties. Results for this are either based on an elastic continuum (Gaspar et al., 2003) or on a material with microstructure (Koenders, 2005).

The first two mechanisms can be considered to be the dual of each other. This work is concerned with the practical realisation of a material with a negative Poisson's ratio via the second mechanism.

2. Isotropic, homogeneous material

Bathurst and Rothenburg formulate the incremental response of an assembly of elastic spheres from an isotropic distribution of contacts around a particle. At the heart is a linear elastic model for the contact between two spheres:

$$f_n = k_n \delta_n, \tag{1}$$
$$f_t = k_t \delta_t, \tag{2}$$

where $f_\bullet$ is the force between two contacting spheres due to relative displacement $\delta_\bullet$ of their centres. The parameters k_n and k_t are then the normal and tangential linear elastic constants and the interaction ratio λ, is defined by $\lambda = k_t/k_n$. Jenkins and Koenders (2004) follow a similar process that encompasses Bathurst and Rothenburg's results. Using their notation, the incremental response between stress, σ_{pk}, and mean strain, α_{st}, of a homogeneous assembly of spherical particles is:

$$\sigma_{pk} = Z_{pkst}\alpha_{st}, \tag{3}$$
$$Z_{pkst} = \frac{1}{2v}\left(A_{pstk} - \varepsilon_{nqr}\varepsilon_{ij\ell}A_{pijk}(\mathbf{B})^{-1}_{\ell n}A_{rstq}\right), \tag{4}$$
$$B_{pk} = \varepsilon_{pqr}\varepsilon_{ijl}A_{rijq}, \tag{5}$$
$$A_{rijq} = D^2 \sum_{\text{neighbours}} \left(k^n n_r n_i + k^t t_r t_i\right) n_j n_q. \tag{6}$$

Variables are defined in Table 1.

The first term on the right hand side of Eq. (4) is Bathurst and Rothenburg's result. The second term on the right hand side incorporates rotation of the spheres. The tensor **Z** is therefore an estimate of the incremental elastic moduli of the assembly of spheres. From these calculations, neglecting the rotation term, the Poisson's ratio of such an assembly in 2D and 3D, respectively is:

$$v = \frac{1-\lambda}{3+\lambda}, \tag{7}$$
$$v = \frac{1-\lambda}{4+\lambda}, \tag{8}$$

which is plotted in Fig. 3. This clearly demonstrates a prediction of a negative Poisson's ratio when λ increases above unity. In 2D, Koenders (2005) examines the relationship between the interaction ratio λ and the relative angles between contacting spheres. He finds that there are no possible isotropic assemblies where $\lambda > 1$. The contradiction to Bathurst and Rothenburg's result is entirely due to incorporating the rotation term in the local equilibrium equations.

Since we no longer have isotropy an internal angle ψ is introduced to distinguish possible orientations. The Poisson's ratio as a function of ψ is then

$$v(\psi) = \frac{(1-\lambda)\sin^2(2\psi)}{4\cos^4(\psi) + \lambda\sin^2(2\psi)}. \tag{9}$$

The Poisson's ratio for values of ψ are plotted in Fig. 4.

Table 1
List of variables.

D	Diameter of spheres
f^n, f^t	Normal and tangential forces between spheres
G	Shear modulus of sphere material
k^n, k^t	Normal and tangential elastic interactions
n	Unit normal vector between sphere centres
N	Mean number of contacts per sphere
t	Unit vector tangential to contact between spheres
v	Volume of assembly of spheres
α_{ij}	Mean bulk scale strain increment
ε_{ijk}	The permutation tensor
ν	Poisson's ratio of bulk assembly of spheres
ν_G	Poisson's ratio of sphere material
σ_{ij}	Bulk scale stress increment