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Force chains and resonant behavior in bending of a granular layer on an elastic support

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ABSTRACT

In this paper we investigate the interaction between a granular layer and an elastic foundation using a coupled Discrete Element Method-Finite Element Method (DEM–FEM) computational model. We use this dynamics code to simulate quasi-static bending of the granular layer and we observe the changes taking place in the structure of the force chains for two cases: with and without rolling resistance. A reversal of the arches formed in the force chains leads to a bending resistance similar to that observed in dynamic experiments on resonant behavior under bending of a layer of sand in a container with an elastic bottom. The behavior of the force chains generated during bending may lead to enhanced mixing in vibrated granular media. In free vibration, the granular-layer/elastic-beam system behaves like a mass-loaded beam with no contribution to the stiffness from the granular layer, independent from the layer thickness and rolling resistance. This is observed to be due to the absence of the reversal of the force chain structures noticed in the quasi-static case when a push-up force bends the system upwards and the force chains rest against the middle of the beam and the side walls. Future studies are required for explaining the experimental observations related to the particle-size dependence of the bending stiffness in a granular layer as well as the resonant behavior of the system under forced bending vibrations.

1. Literature review and problem description

The discrete element method (DEM) has become a very popular tool for studying the micro–macro mechanical behavior of granular materials since first proposed by Cundall and Strack (1979). Applications of the DEM for modeling granular flow, and mixing and segregation, are now covering diverse fields such as powder technology, pharmaceutical industries, food industry and agriculture, geotechnical processes and civil engineering, mining, etc. (e.g., Antony, 2007; Campbell, 1997, 2006; Cleary, 1998; Kudrolli, 2004; Lewis et al., 2005; Onate and Rojek, 2004; Sebastian and Luis, 2005; Tijskens et al., 2003; Yamane, 2004). The DEM is known as a "soft particle" method because the contacting particles are allowed to slightly deform (virtual overlap). The contacting point between particles is taken approximately at the center of the overlap region. In the DEM, the motion of an individual particle in the system is computed as follows: the particles' positions determine the overlap, which results in interaction forces (repulsive and/or attracting based on a particular mechanical model of interaction) that are integrated using the linear and angular momentum balance equations to compute the velocities of the particles. The new particles' positions are then updated using a particular integration scheme.

Early versions of DEM used simplified contact models, such as spring-dashpot (Cundall and Strack, 1979) but since then, other contact force models based on contact mechanics equations developed by, for example, Hertz (1882) for normal forces and Mindlin and Deresiewicz (1953) for tangential forces, have been used (see also Johnson, 1985). The choice for the contact force model depends

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very much on the specific geometry, material properties, and the granular flow characteristics. Among the simplified models, the contact force represented by the linear spring-dashpot model is widely used (for some recent applications with this model see, e.g., Haff and Werner, 1986; Kuo et al., 2002; Cleary and Sawley, 2002; Schafer et al., 1996; Taguchi, 1992; Thompson and Grest, 1991; Zhang and Whiten, 1996). Advantages of this model are that it can be easily developed and applied for various particle shapes and geometries such as spheres, disks, ellipsoids, and the stiffness and damping parameters are analytically derived from the normal restitution coefficient and duration of time in contact (see, e.g., Schafer et al., 1996; Kruggel et al., 2007). Renzo and Maio (2004), however, point out that for this simple model to give reasonable results in simulations of granular flow, the parameters used need to be precisely evaluated. Among the nonlinear contact force models, Hertz theory is extensively used to compute normal forces while the Mindlin–Deresiewicz theory is used for tangential forces. The extension of Hertzian contact for cylindrical particles is given in Poritsky (1950) for cylinders and in Gerl and Zippelius (1999) for disks in 2D. Comparisons between using a contact-based model and a simplified model in modeling the mechanics of granular matter are reported in e.g., Renzo and Maio (2004), Ji and Shen (2006). The nonlinear contact models have been used in, for example, modeling granular flow in a hopper (Langston et al., 1994), heap formation (Baxter et al., 1997), shot peening processes (Han et al., 2000a,b), contact of granular particle system to quantify inter-particle forces, velocity, and contact stresses (Thornton and Randall, 1988), impact of spherical particles with and without adhesion (Thornton and Yin, 1991).

The combined Discrete Element Method-Finite Element Method (DEM–FEM) was first proposed to study shot peening process by Petrinic (1996) in his doctorate work. Han et al. (2000a) gives results for 2D simulation of shot peening and explains in detail the treatment of possible contacts between a disk and the line segments of the finite elements. The shot is modeled by a discrete element while the impacted surface is modeled with finite elements. A review and equivalencies between various contact force models for small deformations is included. An extension of the model to 3D for shoot-peening applications is published in Han et al. (2000b). The combined DEM–FEM in 2D dynamic analysis of geomechanical problems is studied in Onate and Rojek (2004). This study involves fracture in cohesive granular material and plastic flow and wear in a cutting tool. Several examples are shown simulating rock cutting and tool wear, strip punch test and soil, and pipe interaction leading to pipe ovalization. The cutting tool is modeled by finite elements first, to simulate the plastic deformation, and then by discrete elements to model the wearing process. The soil or rock samples are modeled using discrete elements. Other versions of coupling between FEM and DEM are used in applications for reduced models of concrete structures in impact problems (see Frangin et al., 2006), introducing deformability in DEM particles for impact problems (Komodromos and Williams, 2004; Komodromos, 2005) and flow and compaction of irregular, randomly packed, particles to form a tabletted product (Gethin et al., 2006).

To the best of the authors' knowledge, there are no publications regarding the use of coupled DEM–FEM method to analyze the behavior of the force chains and the resonant frequencies of a granular-layer/elastic-beam system in bending deformation and vibration. In this paper we study the interaction (static and dynamic bending deformations) between a granular layer of disks loaded on top of a compliant elastic beam. The behavior of granular materials on vibrating plates has important applications in landmine detection (see e.g. Kang, 2006; Kang et al., 2007, and references therein). The problems studied in the present contribution are dimensionally-reduced versions of the system used in the experiments reported in Kang (2006) and Kang et al. (2007). In addition, recent simulation results (Promratana, 2008) have shown that shaking (in a container with elastic bottom) combined with forced vibration of the elastic bottom of the container can lead to dramatic enhancement of mixing and/or segregation in granular materials when compared to shaking in containers with inflexible bottoms.

The subsequent sections in this paper are arranged as follows: the equations of motion and the force model used in our simulations are described in Section 2; the 2D coupled DEM–FEM model for the granular-layer/elastic-beam system is given in Section 3. The 2D coupled DEM–FEM implementation is validated with an ABAQUS FEM model in Section 4. In Section 5 we study the changes in the structures of the force chains and the resonance behavior of a granular-layer/elastic-beam (GLEB) system under slow-dynamic (quasi-static) bending deformation and under dynamic free-vibrations. Section 6 contains the conclusions and plans for future work.

2. Equations of motion and the force model description in the 2D DEM model

The governing equations of motion for a particle i, consisting of translational and rotational motions are described by the linear and angular momentum balance equations:

$$m_i \frac{d\vec{v}_i}{dt} = \sum_j (\vec{F}^n_{ij} + \vec{F}^t_{ij}) + m_i\vec{g} = \vec{F}^n_i + \vec{F}^t_i + m_i\vec{g} \tag{1}$$

$$I_i \frac{d\vec{\omega}_i}{dt} = \sum_j (\vec{R}^c_{ij} \times \vec{F}^t_{ij} - \vec{M}^r_{ij}) = \sum_j (\vec{R}^c_{ij} \times \vec{F}^t_{ij}) - \vec{M}^r_i \tag{2}$$

where $\vec{v}_i$ and $\vec{\omega}_i$ are the translational and rotational velocity vectors of particle i; m_i and I_i are mass and moment of inertia of particle i; $\vec{F}^n_i$ and $\vec{F}^t_i$ are total normal and tangential forces acting on particle i due to contact particle j; $m_i\vec{g}$ is the body force acting on particle i; $\vec{R}^c_{ij}$ is the vector pointing from the contact point of the contacting pair (i,j) to the center of particle i with its magnitude equal to the particle radius R_i; $\vec{M}^r_i$ is the total resisting moment acting on particle i and caused by rolling friction. A schematic of forces acting on particle i from particle–particle contact interaction is shown in Fig. 1-left.