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Thickness effect on the contact behavior of a composite laminate indented by a rigid sphere

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Abstract

A modified Hertz contact law is proposed for the contact problem of a laminated plate indented by a rigid sphere, which involves the effects of the thickness, in-plane dimensions and boundary conditions of the plate. The model shows that the deflection difference between the center and the edge of the contact area is a key factor dominating the effects of these variables on the force–indentation response. The modified contact law is assumed to follow the same mathematical formulation as the Hertz law. However, the contact stiffness is no longer constant but varies as a function of the contact force due to the thickness effect. The predictions of the force–indentation response show improved agreement over early work by Yang and Sun. Over the present analysis range, the thickness effect may be negligible if the plate thickness is greater than 2 mm. On the other hand, the thickness has a more significant effect on the force–indentation response if the plate thickness is less than 2 mm. Moreover, the in-plane dimensions and the boundary conditions of the plate show little influence on the force–indentation response within the present scope of analysis.

Keywords: Composite; Contact; Indentation; Contact stiffness; Thickness effect

1. Introduction

The damage caused by an impact event is typically more severe for laminated composite structures. In the analysis of the laminated plate subjected to impact loading, the contact theory is of importance and interest in applications. A variety of solutions is available for the contact problems of isotropic materials and has been summarized (Johnson, 1985). The contact theory proposed by Hertz (Love, 1994) was widely used in the contact problems of homogeneous isotropic bodies and has been still given attention to recently (Liu et al., 2005a,b). Essenburg (1962), and Keer and Miller (1983) have studied the contact problems of homogeneous isotropic thin plates via Hertz theory. However, solutions for non-isotropic materials are much less available. Work on contact loading of transversely isotropic materials has been presented by Green and Zerna (1954), Leknitskii (1963), Sveklo (1964), Dahan and Zarka (1977), Conway (Zukas, 1982), and in a particularly convenient form by Turner

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(1966). Shi et al. (2003) analyzed indentation by a rigid ellipsoidal indenter via 3D orthotropic elasticity and proposed a numerical method for solving contact parameters. A procedure for determining some of the features of contact for generally anisotropic materials has been proposed by Willis (1966) that involved numerical contour integration. Even if this approach is more general, its application is difficult because it requires the numerical determination of some parameters. Recently, Swanson (2004) examined the problem of contact loading of an orthotropic half-space and indicated that the procedure outlined by Willis (1966) can be readily employed, along with iteration, to determine several features of the contact analysis, such as the size of the elliptical contact area, the contact pressure distribution, and the indentation depth. The above work is for contact of a half-space only.

One of the most widely used contact laws for laminated plates is that proposed based on Hertz theory by Yang and Sun (1983). In Yang and Sun's work, it is assumed that the contact pressure and contact area could be obtained from the usual formulas for isotropic materials, but with the isotropic modulus of elasticity replaced by the orthotropic modulus in the loading direction. It has been indicated that this model does not take into account the plate thickness effect. For small indentations, Wu and Shyu (1993) reported good agreement between experimental results and the predictions of Yang and Sun's model. However, when comparing predictions with the experimental results obtained by Tan and Sun (1985), it can be seen that there is considerable deviation for relatively large dent depths. To approximate the contact problem of anisotropic plates, the method generally used is to combine an elasticity solution which describes the local contact phenomenon and a classical theory for the global response. For example, Cairns and Lagace (1987) have used the stress function proposed by Leknitskii (1963) to study the thick composite laminates subjected to lateral loading. In their study, a Hertz pressure is assumed and the resolution uses Bessel–Fourier's functions. Their experimental results were found to be in good agreement with their predictions. On the other hand, some researchers use Green's functions and integral equations to correlate the contact force and the plate response. For example, Sankar (1989) derived an approximate Green's function for surface displacement in an orthotropic beam which is used to formulate the integral equation for the problem of smooth contact between a rigid cylinder and an orthotropic beam. Wu and Yen (1994) and Chao and Tu (1999) have used Pagano's solution as a Green's function approximating a point load and then numerically associated the resulting surface displacements to the indentor geometry. With such an approach they related the static indentation of a cross-ply laminate to the contact force exerted by a rigid sphere. As indicated by Swanson (2005), however, it is difficult to assess the resulting accuracy.

As reviewed above, the contact problems of laminated plates have been investigated with the modified Hertz theory or by numerical techniques based on elastic solutions. It is noted that the modified Hertz law (Yang and Sun, 1983) is unable to relate the contact force to its indentation accurately for laminated composites of finite thickness. In particular, its predictions deviate considerably from experimental results for relatively large dent depths. It is reasonable to ascribe such a deviation to the fact that the Hertz theory was originally developed for an elastic half-space and thus it has limited applicability to finite thickness anisotropic plates. Until now, no further studies of contact of the laminated plates have been presented in the literature as noted by Swanson (2005). The present study proposes a method for the contact problem between a laminated composite plate and a rigid sphere. The method allows contact stiffness vary with load, provided that the classical Hertz contact law may be modified by introducing a geometrical relationship for deformation in the contact region. The effects of the thickness, in-plane dimensions, and the boundary conditions on the force–indentation response of the laminates can be included in the analysis by the variable stiffness based contact law.

2. Contact law for layered half-space composites

Yang and Sun (1983) proposed an approximation for the contact deformation of a composite laminate indented by a rigid sphere, given as follows

$$F = k\alpha^{1.5} \tag{1}$$

where α is the contact deformation, F is the contact force and k is the Hertzian contact stiffness which can be approximated by the expression

$$k = \frac{4}{3}\sqrt{R}E_3 \tag{2}$$