Available online at www.sciencedirect.com

ScienceDirect

Mechanics of Materials 39 (2007) 1006–1024

MECHANICS OF MATERIALS

www.elsevier.com/locate/mechmat

Effective poroelastic properties of transversely isotropic rock-like composites with arbitrarily oriented ellipsoidal inclusions

A. Giraud [a,b,*], Q.V. Huynh [b], D. Hoxha [b], D. Kondo [c]

[a] *LPMM-UMR CNRS 7554/ISGMP, Université Paul Verlaine-Metz, Ile du Saulcy, 57045, Metz Cedex 01, France*
[b] *LAEGO-ENSG, BP40, Vandoeuvre-lès-Nancy Cedex F-54501, France*
[c] *LML-UMR CNRS 8107, USTL, cité scientifique, boulevard Paul-Langevin, 59655 Villeneuve d'Ascq cedex, France*

Received 6 October 2006; received in revised form 4 May 2007

Abstract

The present work is devoted to the determination of the macroscopic poroelastic properties of transversely isotropic geomaterials or rock-like composites with arbitrarily oriented ellipsoïdal inhomogeneities. The key problem to solve is to separate respective effects of matrix anisotropy and of inhomogeneities' orientation distribution and shape. Based on a numerical integration of the exact Green function provided by Pan and Chou [Pan, Y.C., Chou, T.W., 1976. Point force solution for an infinite transversely isotropic solid. Journal of Applied Mechanics 43, 608–612] for the transversely isotropic media, the Hill tensor $\mathbb{P}$ is obtained for an arbitrarily oriented oblate spheroidal inclusion. The integrate of the Hill tensor is performed in an intermediate system coordinate attached to the inhomogeneity and with one axis equal to the symmetry axis of the transversely isotropic matrix. This choice of system coordinate allows to simplify numerical calculations and to gain accuracy because partial integrate can be performed analytically. The obtained Hill tensor is next used to study the effect of matrix anisotropy, pores systems and microstructure-related parameters on the overall effective poroelastic properties in transversely isotropic rocks. As an example, a two step-homogenization scheme is presented to test the ability of the proposed model in the evaluation of effective Biot tensor and Biot modulus and stiffness tensor. With the help of an orientation distribution function (ODF) the anisotropy due to the pore systems is also accounted. Numerical applications are finally carried out for anisotropic porous rocks both composed of a matrix with solid minerals constituents and a pore space.

Keywords: Eshelby tensor; Green function; Transverse isotropy; Ellipsoïdal inclusion; Micromechanics; Biot tensor; Poroelasticity

1. Introduction

This paper treats the estimation of the effective poroelastic properties of rock-like composites by using the Eshelby and Hill polarization tensors for an arbitrarily oriented ellipsoidal inhomogeneity in

* Corresponding author. Address: LAEGO-ENSG, BP40, Vandoeuvre-lès-Nancy Cedex F-54501, France.
E-mail address: Albert.Giraud@univ-metz.fr (A. Giraud).

0167-6636/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mechmat.2007.05.005

a transversely isotropic elastic material. The key problem to solve is to take into account the anisotropy of the matrix and also the anisotropy induced by the pore space orientation distribution. Many papers have been presented for the case of transversely isotropic heterogeneous material with isotropic matrix. In this case, the effective observed anisotropy is only induced by the orientation distribution of the embedded inclusions. The well known Eshelby $\mathbb{S}^E$ and Hill tensors $\mathbb{P}$ for ellipsoidal inclusions in an isotropic medium (Eshelby, 1957; Eshelby, 1961; Mura, 1987; Nemat-Nasser and Hori, 1993) can be used through homogenization schemes. Aligned ellipsoidal inclusions, randomly distributed or oriented distribution orientation of ellipsoidal inclusions in an isotropic matrix can be taken into account in various situations. Recent results have been presented for transversely isotropic rock-like composites in Tod (2003), Jakobsen and Johansen (2005) and Levin and Markov (2005). The case of aligned ellipsoidal inclusions in a transversely isotropic matrix has been also widely studied. The Eshelby $\mathbb{S}^E$ and Hill $\mathbb{P}$ tensors have been established by various authors: see Withers (1989) for the analytical solution of the problem of the ellipsoidal inclusion in a transversely isotropic elastic material. A general solution of this problem has been recently presented by Sevostianov et al. (2005) for various inhomogeneities (including spherical inhomogeneity) in a transversely isotropic material. Kirilyuk and Levchuk (2005) present a solution based on Fourier transform method for the same problem. Homogenization methods based on Eshelby tensors take no difficulty as $\mathbb{S}^E$ and $\mathbb{P}$ are well known.

The problem studied in this paper is more complicated than the two particular cases previously mentioned: the matrix is supposed transversely isotropic and the inclusion orientation distribution may be aligned, random or oriented. This case corresponds to observed situation in rock-like composite material such as argillites and shale: macroscopic properties are transversely isotropic as well as argillaceous matrix, and orientation distribution of pores (modelled as ellipsoidal inclusions) is also transversely isotropic with a different anisotropic degree than the matrix. The fundamental problem to solve is the one of the isolated ellipsoidal inclusion with arbitrarily orientation in an infinite transversely isotropic medium. No analytical results have been provided in the general case of the ellipsoidal inclusion with arbitrarily orientation in an anisotropic solid so numerical methods have to be used to evaluate $\mathbb{S}^E$ and $\mathbb{P}$ tensors. It may be noticed that numerical procedures have been extensively developed for evaluation of Eshelby tensor $\mathbb{S}^E$ and Hill tensor $\mathbb{P}$ in general anisotropic solids (see Suvorov and Dvorak, 2002; Gavazzi and Lagoudas, 1990; Ghahremani, 1977). A powerful and general numerical method is based on Fourier Transforms and Radon transform (see among others Mura, 1987 and more recently Franciosi and Lormand, 2004).

The numerical method used in this paper to obtain the Hill tensor of the isolated arbitrarily oriented ellipsoidal inclusion in a transversely isotropic matrix is based on numerical integration of the exact Green's function provided by Pan and Chou (1976) for the transversely isotropic media. The particular case of the oblate spheroidal is investigated in this paper. By performing the numerical integration in an *intermediate* system coordinate respecting the symmetry of the material, it is then found that the Hill tensor depends only on one angle (the angle between the symmetry axis of the ellipsoid and the symmetry axis of the matrix). The Hill tensor in the fixed global system coordinate of the material is then deduced from simple transformation rules between the *intermediate* and *global* systems coordinate. This method can be used as an alternative to the powerful Fourier transform based methods in the particular case of transverse isotropy. The obtained Hill tensor is then used to evaluate the effective poroelastic properties by homogenization. The obtained results are next used to study the effect of matrix anisotropy, pores systems and microstructure-related parameters on the overall effective poroelastic properties in transversely isotropic rocks. A similar work based on numerical integration of Green's function for the conduction problem has been recently presented in Giraud et al. (2007a).

1.1. Notations and definitions

Hereafter we define some notations and recall some results which are needed later (see among others Nemat-Nasser and Hori, 1993). Barred letters $\mathbb{A}$, $\mathbb{C}$, $\mathbb{D}$, $\mathbb{Q}$ refer to fourth-order tensors, bold-face letters $\boldsymbol{\varepsilon}$, $\boldsymbol{\sigma}$, $\boldsymbol{\delta}$ refer to second-order tensors, underlined letters $\underline{z}$, $\underline{x}$ refer to first order tensors. Einstein's summation convention over repeated indices is used unless otherwise indicated. $\otimes$ and: operators respectively represent dyadic product and contracted product on two indices. $\boldsymbol{\delta}$, $\mathbb{I}$, $\mathbb{J}$ and $\mathbb{K}$ respectively represent the second-order identity tensor, the