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Determination of the constitutive relation and critical condition for the shock compression of cellular solids

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ABSTRACT

This study aims at understanding the constitutive relation and critical condition for the shock compression of cellular solids. A 2D virtual foam is constructed from the cross-section of a closed-cell aluminium foam imaged by micro X-ray computed tomography, which enables the realistic consideration of meso-scale structural effect in numerical modelling. Quasi-static and shock compressions of the 2D foam are simulated. A series of Hugoniot relations between shock speed (and other mechanical quantities) and impact speed are determined from the finite element (FE) simulations. It is found that the shock speed increases approximately linearly with impact speed, similar to that observed for condensed solids, but the related material constants for cellular solids have different physical implications, whereas the shock strain, stress and energy increase with impact speed nonlinearly, due to shock-enhanced cell compaction and cell-wall plastic deformation. Based on conservation laws in continuum mechanics, other Hugoniot relations are derived from the basic linear one, which agree well with those obtained from the FE simulations. It is thus demonstrated that the unique linear Hugoniot relation can be used to characterise the shock constitutive behaviour which is distinct from the quasi-static one. Furthermore, a new analytical method based on the linear Hugoniot relation is proposed to estimate the critical impact speed for shock initiation, which has reasonable agreement with the present FE simulations and previous experimental and numerical results, and outperforms the existing methods.

1. Introduction

Cellular solids are characterised by high porosity (usually exceeding 70%), leading to their distinctive mechanical, thermal, electromagnetic and other properties attractive for various engineering applications (Gibson and Ashby, 1997). Under high speed impact or intensive blast, shock compression can be initiated in cellular solids and the load transmitted can be significantly increased (Elnasri et al., 2007; Li and Meng, 2002; Reid and Peng, 1997; Tan et al., 2012; Tan et al., 2005a), which may enhance the energy absorption but not benefit structural protection. Therefore, it is important to understand the shock behaviour of cellular solids.

Extensive experiments have identified two prominent features of shock compression in cellular solids (Barnes et al., 2014; Radford et al., 2005; Reid and Peng, 1997; Tan et al., 2012; Tan et al., 2005a): (1) a significant enhancement of stress measured at the impact end; and (2) a localisation of cell crushing adjacent to the impact end. Meanwhile, numerical studies on the dynamic crushing of idealised 2D cellular solids have been reported to understand the effects of cell irregularity (Zheng et al., 2005), non-uniform cell-wall thickness (Li et al., 2007), cell micro-topology (Liu and Zhang, 2009) and structural defects (Zhang et al., 2010) on the shock behaviour. However, Sun et al. (2015; 2016b) have demonstrated the importance of using a realistic cell geometry in the simulation of dynamic crushing in order to capture the distinctive shock behaviour. Therefore, it is necessary to apply a more realistic cellular solid in modelling.

"Shock", as one of the deformation modes of cellular solids, is used here as a term for the propagation of the planar interface (i.e. shock front) separating the crushed and uncrushed cells in dynamic compression, which has similar features to shock wave propagation in a condensed solid (Davison, 2008; Meyers, 1994; Wang, 2007). To demonstrate the "shock" feature of the shock front, Zou et al. (2009) performed finite element simulations and numerically observed a jump, or a macroscopic discontinuity, in the key mechanical quantities (i.e. velocity, stress and strain) across the shock front (about one cell size in thickness) of hexagonal-cell

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honeycombs. Liao et al. (2013) numerically confirmed the existence of a shock front for idealised irregular 2D cellular structures.

The measurement of the speed of the shock front (i.e. shock speed for brevity) has also been attempted in experiments and simulations. For instance, Barnes et al. (2014) experimentally observed that the variation of shock speed with impact speed generally follows a linear trend for open-cell aluminium Duocel foam and a linear equation was used to fit the experimental data. Liao et al. (2013) derived the relation between shock speed and impact speed for 2D Voronoi foam based on various idealisations of the quasi-static stress-strain relation; but their analytical predictions showed a marked difference from the numerical result, especially at high impact speeds (see Fig. 13 in Ref. (Liao et al., 2013)). Similarly, Pattofatto et al. (2007) compared the analytical predictions of shock speed derived from the quasi-static stress-strain relation for closed-cell aluminium Alporas foam and they stated that their nonlinear equation (see Eq. 9 in Ref. (Pattofatto et al., 2007)) gave a “satisfactory prediction”. More recently, Zheng et al. (2014) measured the dynamic stress-strain relation for 3D Voronoi foam numerically and recommended a dynamic material model, i.e. Eq. (14) in Ref. (Zheng et al. (2014)), based on which, we found that a linear relation between shock speed and impact speed, i.e. $\dot{\Phi} = v + \sqrt{D/\rho_0}$, can be derived (see Appendix) where $\dot{\Phi}$ is the shock speed, v is the impact speed, ρ_0 is the density and D is a material constant defined therein. The simple linear relation between shock speed and impact speed seems more fundamental for characterising and modelling the shock behaviour of cellular solids. However, it requires more experimental and numerical supports and a better understanding of its physical implications.

On the other hand, increasing efforts have been made to establish the constitutive relation under shock compression. In an early study, Reid and Peng (1997) proposed a shock model based on a rigid, perfectly-plastic, locking (r-p-p-l) material model to analyse the impact response of wood, which was subsequently adopted by other researchers to analyse the shock behaviour of various cellular solids (Li and Reid, 2006; Ma et al., 2009; Ruan et al., 2003; Tan et al., 2005b; Zou et al., 2009). The continuous improvement of this shock model has occured mainly from the adoption of more mathematically realistic constitutive equations such as linear hardening (Zheng et al., 2012), power-law hardening (Zheng et al., 2013) and complicated nonlinear hardening (Karagiozova et al., 2012) equations. However, the basic assumption that the shock property of the material is independent of the local crushing velocity has not been changed. In other words, the quasi-static compression tests were considered to be able to provide complete input parameters for the shock model. Recently this assumption has been proven questionable. For instance, Zheng et al. (2014) employed a 3D Voronoi finite element model to establish a dynamic stress-strain relation which is shown different from the quasi-static one. Barnes et al. (2014) experimentally measured the mechanical variables of the Duocel foam at different impact speeds and demonstrated that a complete description of shock behaviour requires the direct measurement of Hugoniot relations (i.e. the loci of all shocked states) and the material states under shock cannot be determined from the quasi-static stress-strain relation. These recent findings do not support the conclusion made by Pattofatto et al. (2007) that “shock enhancement effect should not be taken into account at the level of the constitutive law itself”.

Equally importantly, previous experimental and numerical studies have shown that there exists a critical impact speed, above which shock occurs (Barnes et al., 2014; Tan et al., 2005a; Zou et al., 2009). For compression at subcritical impact speeds, localised cell crushing occurs in presumably “weak” cells or sites, and the crush bands are randomly distributed and the boundary between crushed and uncrushed cells is not necessarily flat (Barnes et al., 2014; Liu et al., 2009; Sun et al., 2014; Tan et al., 2005a; Zheng et al., 2014), in contrast to the shock deformation mode. The critical condition for shock initiation is of fundamental importance and practical interest. However, it still lacks a recognised analytical method to determine this critical impact speed, and there is confusion about the factors that influence the critical impact speed (Wang et al., 2013).

The objective of this study is to clarify the above outstanding issues through a combination of image-based modelling and continuum-based theory. A 2D virtual foam was created from a computed tomography (CT) image of a cross-section of a closed-cell aluminium Alporas foam sample and a finite element (FE) model with the same meso-scale complexity as the real foam was developed for compression simulations. First, the quasi-static compression of the 2D foam was simulated for comparison purposes. Then, the shock compression was simulated at different impact speeds to obtain the complete Hugoniot relations (e.g. the dependences of shock speed and material states on impact speed, and the stress-strain relation) and to define the shock constitutive relation. Furthermore, a new analytical method based on the basic Hugoniot relation (shock speed vs. impact speed) is proposed to estimate the critical impact speed for shock initiation.

2. Image-based modelling

2.1. Two-dimensional foam

A diametral X-ray computed tomography (CT) slice image of a cylindrical sample (Ø30 × 30 mm) of closed-cell aluminium Alporas foam was used to construct a 2D virtual foam (30 × 30 mm) with realistic cell geometry, as shown in Fig. 1. The CT scanning of the real foam sample was performed in a Nikon Metris CT system housed in a customised bay at the Henry Moseley X-ray Imaging Facility (HMXIF, The University of Manchester). An acceleration voltage of 70 kV, a current of 280 μA, an effective voxel size of 19.1 μm, and an exposure time of 500 ms for each of 2000 projections over 360 degrees were used. The X-ray radiographs were reconstructed using Nikon Metris CT-Pro software into CT images. A greyscale-based segmentation method was then used to extract the cell structure of the foam, using a threshold of grey values to ensure the separation of all the solid parts from the surrounding air.

It is clearly seen from Fig. 1 that the cell morphology and topology of the 2D virtual foam are much more complex than those of the idealised 2D cellular solids used previously such as hexagonal-cell, circular-cell and Voronoi ones (Liu et al., 2009; Sun and Li, 2015; Zheng et al., 2005). Some structural imperfections, e.g. corrugation, bowed or damaged walls, and non-uniform cell-wall thickness, are evident, which are related to the liquid-state foaming process of Alporas foam (Simone and Gibson, 1998a). These kinds of structural defects may play important roles in the determination of macroscopic material properties (Chen et al., 1999; McDonald et al., 2006; Simone and Gibson, 1998b, 1998c). However, due to the difficulty to control the variation of the structural imperfections in the selected image-based foam sample, the influence of structural imperfection will not be a focus of this study.

To take account of realistic geometrical features, continuum elements have to be used in a finite element model (FEM), which dramatically increases the computational expense and may also cause numerical problems due to element distortion. The 3D FE modelling of the compressive behaviour of Alporas foam has been attempted by the same authors (Sun et al., 2016b). However, the 3D FE simulation is still restricted to relatively small samples and less extensive deformations for limited loading cases