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Maximum disorder model for dense steady-state flow of granular materials☆

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ABSTRACT

A flow model is developed for dense shear-driven granular flow. As described in the geomechanics literature, a critical state condition is reached after sufficient shearing beyond an initial static packing. During further shearing at the critical state, the stress, fabric, and density remain nearly constant, even as particles are being continually rearranged. The paper proposes a predictive framework for critical state flow, viewing it as a condition of maximum disorder at the micro-scale. The flow model is constructed in a two-dimensional setting from the probability density of the motions, forces, and orientations of inter-particle contacts. Constraints are applied to this probability density: constant mean stress, constant volume, consistency of the contact dissipation rate with the stress work, and the fraction of sliding contacts. The differential form of Shannon entropy, a measure of disorder, is applied to the density, and the Jaynes formalism is used to find the density of maximum disorder in the underlying phase space. The resulting distributions of contact force, movement, and orientation are compared with two-dimensional DEM simulations of biaxial compression. The model favorably predicts anisotropies of the contact orientations, contact forces, contact movements, and the orientations of those contacts undergoing slip. The model also predicts the relationships between contact force magnitude and contact motion. The model is an alternative to affine-field descriptions of granular flow.

1. Introduction

The critical state concept in geomechanics holds that dense granular materials, when loaded beyond an initial static packing, eventually attain a steady state condition of constant density, fabric, and stress (Schofield and Wroth, 1968). This condition is often associated with shear-driven flow and failure: granular avalanches, landslides, tectonic faults, and failures of foundation systems and embankments. As such, the critical state has received intense interest from geologists, engineers, and physicists, who have devoted great effort in understanding the state's underlying mechanics. Density, fabric, and deviatoric stress at the critical state are known to depend upon the particles' shapes and contact properties as well as on the mean stress and intermediate principal stress (Zhao and Guo, 2013). Even so, the eventual bulk characteristics for a given assembly are insensitive to the initial particle arrangement and to the stress path that ends in the critical state: for example, materials that are initially either loose or dense eventually arrive at the same density condition after sufficient shearing. This convergent characteristic resembles that of thermal systems that approach an equilibrium condition with sufficient passage of time.

Another pervasive feature of critical state flow is the continual and intense activity of grains at the micro-scale, yet this local tumult produces a monotony in the bulk fabric, stress, and density. Micro-scale activity occurs in three

☆ The paper is dedicated to the memory of Dr. Masao Satake (1927–2013), who made significant fundamental contributions to granular mechanics.

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ways: (a) statically, as alterations of the inter-particle contact forces, (b) geometrically, as changes in the particles' configuration and local density, and (c) topologically, as changes in the load-bearing contact network among the particles. In these three respects, we view the critical state as the condition of maximum disorder that emerges during sustained shearing. Early work by Brown et al. (2000) investigated disorder in the local density, and a recent paper by the author explored topological disorder at the critical state (Kuhn, 2014). The current paper addresses statical disorder as expressed in a two-dimensional (2D) setting, focusing on anisotropies and distributions of the contacts' orientations, forces, and movements. The analysis applies to the critical state flow of dry unbonded frictional materials of sufficient density to develop a load-bearing (persistently jammed) network of contacts during slow (quasi-static, non-collisional) shearing. Particles are assumed durable (non-breaking) and nearly rigid, such that deformations of the particles are small, even in the vicinity of their contacts.

During flow, granular materials have an internal organization of movement and force, an organization that is in some respects pronounced but in others subtle. We briefly review these characteristics of the critical state, as observed in laboratory experiments, numerical simulations, or both.

- A.1. The motions of individual particles do not conform to an affine, mean deformation field, and fluctuations from the mean field are large and seemingly erratic (Kuhn, 2003). The rates at which contacting particles approach or withdraw from each other (i.e. contact movements in their normal directions) are generally much smaller than those corresponding to an affine field. In contrast, the transverse, tangential movements between contacting pairs are much larger than those of affine deformation (Kuhn, 2003). As a result, bulk deformation is almost entirely attributed to the tangential movements of particles (Kuhn and Bagi, 2004).
- A.2. Strength, expressed as a ratio of the principal stresses, is insensitive to the contacts' elastic stiffness and to the mean stress, such that simulations of either soft or hard particles exhibit similar strengths at the critical state (Härtl and Ooi, 2008; Kruyt and Rothenburg, 2014). Fabric measures at the critical state (fraction of sliding contacts, contact anisotropy, etc.) are also insensitive to contact stiffness (da Cruz et al., 2005).
- A.3. Particle rotations are large when compared with the bulk deformation rate (Kuhn and Bagi, 2004). In particular, the rolling motions between particles are much larger than the sliding movements (Kuhn, 2004b).
- A.4. The contact network is relatively sparse, in that the number of contacts within the load-bearing contact network is sufficient to produce a static indeterminacy (hyperstaticity) but with only a modest excess of contacts (Thornton, 2000). The excess very nearly corresponds to the number of contacts that are sliding (Kruyt and Antony, 2007). This modest indeterminacy is consistent with observations of intermittent, sudden reductions of stress, which result from periodic collapse events that are occasioned by fresh slip events or loss of contacts (Peña et al., 2008). This condition of marginal hyperstaticity is referred to as "jammed" within the granular physics community.

- B.1. During critical state flow, the contact fabric is anisotropic, with the normals of the contacts oriented predominantly in the direction of the major principal compressive stress (Rothenburg and Bathurst, 1989).
- B.2. The normal contact forces are larger among those contacts oriented in the direction of the major principal compressive stress; whereas, the averaged tangential forces are larger for contact surfaces oblique to the principal stress directions (Rothenburg and Bathurst, 1989; Majmudar and Bhehringer, 2005).
- B.3. Deviatoric stress is primarily borne by the normal contact forces between particles; whereas, the tangential contact forces make a much smaller contribution to the deviatoric stress (Thornton, 2000).
- B.4. When considering only the normal forces, deviatoric stress is primarily carried by those contacts with forces that are larger than the mean force (strong contacts), whereas the remaining (weak) contacts contribute far less to the deviatoric stress (Radjai et al., 1998; Kruyt and Antony, 2007).
- B.5. Anisotropy of the contact network is also largely attributed to strong contacts, which are predominantly oriented in the direction of the major principal stress (Radjai et al., 1998).
- B.6. Many contacts slide in the "wrong direction" with respect to the direction that corresponds to an affine deformation (Kuhn, 2003).
- B.7. Compared with other orientations, contacts that are oriented in the direction of extension have a greater average *magnitude* of slip, but the mean slip velocity is largest among contacts that are oriented obliquely to the directions of compression and extension (Kuhn and Bagi, 2004).
- B.8. Frictional sliding is more common among those contacts with a smaller-than-mean normal force (Radjai et al., 1998).
- B.9. The more mobile contacts — those with large sliding movements — tend to be those that bear a smaller-than-average normal force (i.e., weak contacts) (Kruyt and Antony, 2007).
- B.10. Deformation, when measured at the meso-scale of particle clusters, is related to contact orientation: contacts with branch vectors that are more aligned with that of bulk compression tend to produce local dilation; whereas, contacts that are more aligned with the direction of bulk extension tend to produce local compression. These trends have been determined by studying the elongations of voids that are surrounded by rings of particles and their branch vectors (Nguyen et al., 2009).
- B.11. The probability density of the normal contact forces usually decreases exponentially for forces that are greater than the mean (Majmudar and Bhehringer, 2005). With forces less than the mean, however, the density is more uniform than exponential.
- B.12. Strength at the critical state increases with an increasing inter-granular friction coefficient, but the relation is non-linear, and little strength gain occurs when