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# Constitutive equations optimized for determining strengths of metallic alloys

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ABSTRACT

We investigate compatibilities of three constitutive equations, the Hollomon, the Swift, and the Voce equations for determination of yield and ultimate tensile strengths based on tensile true stress–strain curves of 27 metal alloys including those with power-law type and linear-type strain-hardening. We analyze each constitutive equation in terms of yield strength determined by the intercept of the linear elastic loading curve and plastic flow curve and ultimate tensile strength evaluated by the concept of instability in tension. We found that the describing plastic flow is very sensitive in determination of the yield strength and tensile strength from parameters of constitutive equation. Voce equation gives estimate yield strength and tensile strength better than Hollomon and Swift equations.



## 1. Introduction

Tensile testing generally produces uniaxial stress–strain relations of materials that are useful enough to predict multi-axial elasto-plastic deformation through conventional mechanics models (Dieter, 1961; von Mises, 1913). The stress–strain relation in the plastic flow region is described by a constitutive equation containing some number of parameters corresponding to tensile properties. The most commonly used constitutive equation is the Hollomon equation, which has the form of a power-law relation between true stress $\sigma$ and strain $\varepsilon$;

$$\sigma = K\varepsilon^n, \tag{1}$$

where $K$ is the strength coefficient and $n$ is the strain-hardening exponent (Dieter, 1961; Hollomon, 1945; Kleemola and Nieminen, 1974). The yield strength can be determined as the intercept of elastic linear line and plastic flow curve described by the constitutive equation (Dieter, 1961; ASTM, 2002). If the plastic flow curve is well described by Eq. (1), the yield strength can be determined using the elastic modulus $E$, $K$ and $n$. The ultimate tensile strength $(\varepsilon_u)(\sigma_u)$ at which necking initiates can be determined using the instability conditions for the Hollomon equation:

$$\frac{\partial\sigma}{\partial\varepsilon} = \sigma, \tag{2}$$

$$\varepsilon_u = n, \tag{3}$$

$$\sigma_u = K\varepsilon_u^n = Kn^n, \tag{4}$$

where $\varepsilon_u$ is the strain at ultimate tensile stress (Dieter, 1961; Kim et al., 2006a). From the equations above, we can determine the yield strength and the ultimate tensile

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strength analytically for materials well described by the Hollomon equation if $E$, $K$, and $n$ are known.

Though yield and ultimate tensile strengths are directly measured in uniaxial tensile testing, constitutive equations play an important role in evaluating tensile stress–strain curves from other indirect methods such as instrumented indentation testing and small punch testing (Kim et al., 2006a; Ahn and Kwon, 2001; Kim et al., 2006b; Buclille et al., 2003; Dao et al., 2001; Lee et al., 2010, 2008; Foulds et al., 1998; Fleury and Ha, 1998; Guillemot et al., 2012). In these tests, flow stress–strain points are not measured for the whole range of the tensile curve, and the ultimate tensile strength cannot be measured due to absence of necking. Thus, tensile properties, yield and ultimate tensile strengths, $K$ and $n$ must be determined using constitutive equations in an analytical way (Kim et al., 2006a; Ahn and Kwon, 2001; Kim et al., 2006b; Lee et al., 2008). The optimum choice and accuracy of constitutive equation is critical in determining tensile strengths in such tests. In addition to the Hollomon equation, other constitutive equations (Yoo and Park, 2008; Swift, 1952; Ludwigson, 1971; Samuel and Rodriguez, 2005; Voce, 1948, 1955; Kim et al., 2013) have been suggested to describe some materials which do not show power-law relation (i.e. austenitic stainless steels show linear-type strain hardening (Lee et al., 2008). Swift and Voce equations are also well known for constitutive equation describing plastic hardening behaviors of metal, where the Swift equation is given by

$$\sigma = K_s(\varepsilon + \varepsilon_0)^{n_s}; \quad (5)$$

the Voce equation is

$$\sigma = \sigma_0 - A\sigma_0 \exp(-\beta\varepsilon). \quad (6)$$

$K_s$, $\varepsilon_0$ $n_s$ are constants for Swift equation and $\sigma_0$, $A$, $\beta$ are for Voce equation. Here, we suggest the optimal among three constitutive equations, the Hollomon, the Swift and the Voce equations, for seventeen metallic alloys with power-law hardening and ten alloys with linear hardening in terms of analytical determination of tensile strengths.

## 2. Experiments

Cylindrical tensile samples with gauge length 25 mm and diameter 6 mm were prepared for 27 metal alloys, SKH51, SK4, SCM21, SCM4, API X100, API X120 (carbon steels), STS403, STS410, STS420J2, STS440 (ferrite-based stainless steels), STS303, STS304, STS304L, STS310S, STS316, STS316L, STS321, STS347 (austenite-based stainless steels), Inconel 600, Inconel 825 (Ni alloys), Ti–5Al–2.5Sn, Ti–6Al–4V, Ti–6Al–6V–2Sn, Ti–7Al–4Mo (Ti alloys), Al2024, Al6061, and Al7075 (Al alloys). Uniaxial tensile tests were carried out using Instron 5582 (Instron Inc., USA) at cross-head speed 1 mm/min as per the ASTM standard (ASTM, 2002). Measured true stress and strain data were analyzed using the commercial software OriginPro 7.5 SR0 (OriginLab Co., MA). Power-law type equations were used for the Hollomon and Swift equations and exponential-type equations for the Voce equation. The data fitting was performed with least square regression and data points are

**Table 1**
Tensile properties; elastic modulus was measured by ultrasonic, yield strength was measured by the 0.2% offset method, ultimate tensile strength was measured by maximum stress point in engineering stress–strain curve, and $n$ and $K$ were evaluated by the Hollomon equation.

| Sample | Elastic modulus (GPa) | Yield | | Ultimate tensile | |
|---|---|---|---|---|---|
| | | Strain | Stress (MPa) | Strain | Stress (MPa) |
| SKH51 (tempered) | 223 | 0.0033 | 280 | 0.1181 | 785 |
| SK4 (tempered) | 204 | 0.0040 | 409 | 0.1638 | 750 |
| SCM21 (tempered) | 194 | 0.0034 | 275 | 0.1476 | 649 |
| SCM4 (tempered) | 177 | 0.0060 | 716 | 0.0667 | 999 |
| API X100 | 203 | 0.0049 | 592 | 0.0891 | 913 |
| API X120 | 210 | 0.0057 | 749 | 0.0532 | 1025 |
| STS403 (tempered) | 211 | 0.0036 | 330 | 0.1563 | 674 |
| STS410 (tempered) | 215 | 0.0037 | 357 | 0.1521 | 672 |
| STS420J2 (tempered) | 211 | 0.0038 | 392 | 0.1305 | 806 |
| STS440 (annealed) | 220 | 0.0035 | 323 | 0.1167 | 817 |
| STS303 (annealed) | 206 | 0.0036 | 314 | 0.4822 | 1082 |
| STS304 (annealed) | 190 | 0.0036 | 311 | 0.5374 | 1163 |
| STS304L (annealed) | 203 | 0.0037 | 338 | 0.4900 | 1116 |
| STS310S (annealed) | 192 | 0.0034 | 258 | 0.3468 | 780 |
| STS316 (annealed) | 198 | 0.0034 | 282 | 0.5014 | 1061 |
| STS316L (annealed) | 198 | 0.0035 | 305 | 0.4524 | 949 |
| STS321 (annealed) | 197 | 0.0036 | 305 | 0.4512 | 911 |
| STS347 (solid solution heat treatment) | 200 | 0.0031 | 211 | 0.4169 | 884 |
| Inconel 600 (annealed) | 170 | 0.0044 | 405 | 0.2715 | 985 |
| Inconel 825 (annealed) | 161 | 0.0041 | 341 | 0.3097 | 984 |
| Ti–5Al–2.5Sn (solution treatment) | 130 | 0.0086 | 863 | 0.0903 | 1029 |
| Ti–6Al–4V (solution treatment) | 110 | 0.0104 | 930 | 0.0858 | 1097 |
| Ti–6Al–6V–2Sn (solution treatment) | 122 | 0.0102 | 998 | 0.1070 | 1174 |
| Ti–7Al–4Mo (solution treatment) | 132 | 0.0098 | 1031 | 0.0802 | 1134 |
| Al2024 (tempered) | 72 | 0.0084 | 461 | 0.1283 | 670 |
| Al6061 (tempered) | 70 | 0.0057 | 259 | 0.0508 | 298 |
| Al7075 (tempered) | 71 | 0.0094 | 524 | 0.0828 | 626 |