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Mechanical properties of hierarchical lattices

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ABSTRACT

This paper focuses on the stiffness and strength of lattices with multiple hierarchical levels. We examine two-dimensional and three-dimensional lattices with up to three levels of structural hierarchy. At each level, the topology and the orientation of the lattice are prescribed, while the relative density is varied over a defined range. The properties of selected hierarchical lattices are obtained via a multiscale approach applied iteratively at each hierarchical level. The results help to quantify the effect that multiple orders of structural hierarchy produces on stretching and bending dominated lattices. Material charts for the macroscopic stiffness and strength illustrate how the property range of the lattices can expand as subsequent levels of hierarchy are added. The charts help to gain insight into the structural benefit that multiple hierarchies can impart to the macroscopic performance of a lattice.

1. Introduction

Materials with a hierarchical microstructure are very common in Nature and are a remarkable source of inspiration for the development of new materials. Wood at the macroscopic level, for instance, can be loosely described as made of an arrangement of hollow tubes, whose walls have a microstructure of hemicellulose reinforced with lignin (Fratzl and Weinkamer, 2007). In bone, up to seven orders of hierarchical organization can be identified, each with a defined structural architecture. At the larger length scale we have the trabeculae, which make up the cancellous bone; the trabeculae are made of a network of osteons, which in turn are made of porous hollow fibres, each consisting of protein fibrils (Rho et al., 1998; Weiner and Wagner, 1998). It is proven that nesting multiple hierarchical levels confers significant benefits to the mechanical properties of biological materials (Chen et al., 2008; Koch et al., 2009; Meyers et al., 2008; Weinkamer and Fratzl, 2011; Gibson, 2012). Structural hierarchy in biological materials is the result of a lengthy optimization process, through which the material is constantly prompted by the natural environment to simultaneously fulfil a broad range of multifunctional and conflicting requirements (Koch et al., 2009; Fleck et al., 2010). In wood, the cellular tissue permits the circulation of vital fluids, and confers high compliance and strength to each organ of the plant. The trabecular structure of bones allows the continuous regeneration and maintenance of the structure, while bearing the operational loads. Nacre, the material of seashells and turtle shells, is made of a complex multi-layered arrangement of calcium carbonate tablets, submerged in a soft organic matrix. It has been demonstrated that the exceptional toughness of Nacre, which far exceeds that of its constituents, is controlled by the architecture of its microstructure (Barthelat, 1861; Espinosa et al., 2009). The high toughness of Nacre is crucial to protect the soft organisms enclosed in the shell, and to allow the growth of the shield.

Whereas environmental constraints guide the adaptive process of material formation over millions of years, engineers can resort to additive manufacturing and nanotechnology to build, in a fairly short time frame and at affordable cost, advanced materials with multiple orders

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of microstructural organization (Barthelat, 1861; Fratzl and Weinkamer, 2007). The latest advance in additive manufacturing has driven recent research to the understanding of the properties of hierarchical materials (Yang et al., 2002; Stampfl et al., 2004; Gaytan et al., 2009; Parthasarathy et al., 2010; Ramirez et al., 2011). The concept of structural hierarchy has been exploited in engineering for a long time, one notable example being the Eiffel tower, a third order hierarchical structure whose relative density, the ratio between the volume occupied by the structure and the volume occupied by the solid material, is just 1.2×10^{-3} (Lakes, 1993). In the literature, a seminal work on materials with structural hierarchy is the one by Lakes (1993), who examined a set of natural and artificial hierarchical materials. In this work, Lakes first proposed a compact expression for the stiffness and strength of materials with isotropic structure at each hierarchical level. Parkhouse (1984) also showed that the process of sub-structuring can be recursively applied to each element of a macrostructure, thus no clear distinction exists, in principle, between structure and material (Parkhouse, 1984). The effect of material heterogeneity, which occurs by structuring a material at multiple length scale with properties that are dissimilar from one order of scale to another, was also studied by Yao et al. (2011). In an experimental and numerical work on the cortical bone of a bovine, Yao et al. (2011) illustrated the benefit that structural hierarchy generates in reducing stress concentration at the nanoscale, as well in improving strength and energy dissipation. Sen and Buehler (2011) showed that structural hierarchy enhances toughness and resistance to crack propagation in brittle materials without the need to introduce additional materials. More recently, Fleck et al. (2010) suggested that materials with multiple orders of structural hierarchy have the potential to further improve the performance of lattice materials, in particular to yield higher stiffness, strength and fracture toughness at lower density. Sandwich panels with hierarchical cores have been also the object of recent investigations (Wadley, 1838; Kooistra et al., 2007). From these works, it emerges that for a given density the strength of a panel with two levels of hierarchy in its core can be up to 12.5 times higher than the strength of a panel with a core with a single hierarchical order. Another example is the work of Zhao et al. (2012), who designed, manufactured and tested a hierarchical woven lattice composite. The lattice walls were made of a woven textile sandwich composite, and at the highest level, three lattice topologies were considered: the square, the triangular and the Kagome lattice. It was shown that the presence of a level of hierarchy in the lattice elements significantly enhances the capacity of the lattice to absorb energy. In a more recent study, Torrents et al. (2012) manufactured and tested a nickel-based microlattice materials with three orders of structural hierarchy from the nanometre to the millimetre scale, and relative density in the range 1×10^{-4}–8.5×10^{-1}. A macroscopic stiffness and strength of one order of magnitude larger than those of existing materials were observed in the lowest relative density range, and were attributed to the existence of multiple hierarchical levels. In another recent work, Rayneau-Kirkhope et al. (2012) applied fractal theory to design beams with multiple levels of hierarchy, thereby obtaining improved buckling strength to mass ratio. The authors also manufactured a beam with two levels of hierarchy by means of rapid prototyping to validate the theoretical results.

In this paper, we use a multiscale approach to quantify the effect of multiple structural hierarchies on a material with a lattice architecture. We show that by nesting multiple levels of lattice hierarchies, and by varying the relative density at each level, the property design space of the solid material can be expanded to reach unexplored areas of the material charts. In the first part of the paper, we examine the stiffness and strength of four planar lattices with high relative density. As expected, when the relative density of all levels tends to unity, the overall properties converge to those of the solid material. The results show that the lattice topology has a strong impact on the overall properties of the material. Bending dominated lattices tend to gain more benefit from the existence of multiple hierarchies, thereby increasing significantly the specific stiffness. Stretching dominated lattices, on the other hand, have already an optimal configuration with respect to stiffness, and thus do not show a major improvement. With respect to plastic yielding, a detriment of the overall material strength is observed in high density lattices due to the recursive effect of stress concentration that occurs at each hierarchical level. In the second part of the paper, we analyse four three-dimensional lattices with both open and closed cells, and examine the stiffness and buckling strength of the material.

2. The multiscale scheme

As an example of a hierarchical lattice, consider the planar structure shown in Fig. 1. At the topmost level, $h - 3$, the lattice has a hexagonal topology and the material of the struts is made of a Kagome lattice, which is the hierarchical level 2. The struts of the lattice at level 2 hold another level of substructure, where the material consists of a square lattice. At the level 1 of the hierarchy, the lattice is made of a uniform solid material, level 0. In this paper, we are interested in describing how the properties of the lattice at the top level change if the number of hierarchical orders and geometrical parameters of the lattices vary at each level. While the relative density of a hierarchical lattice is simply the product of the relative densities at each level, this does not hold for stiffness, strength and other properties. For example, the stiffness of the lattice at a given level is governed by the lattice topology, the geometrical parameters of the unit cell at that level, and the properties of the solid material. Only if we prescribe an isotropic solid material and an isotropic topology at each order, the resulting lattice displays isotropic macroscopic properties. In this case, we can resort to the compact expressions proposed by Lakes (1993) for the Young's modulus and the strength. However in the general case, when the parameters of the lattice at each level are dissimilar, it is necessary to follow a bottom up approach starting from the solid material level, and derive the properties of