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A finite deformation model for evolving flow anisotropy with distortional hardening including experimental validation

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ABSTRACT

The paper presents a new constitutive model for anisotropic metal plasticity that takes into account the expansion or contraction (isotropic hardening), translation (kinematic hardening) and change of shape (distortional hardening) of the yield surface. The experimentally observed region of high curvature (“nose”) on the yield surface in the loading direction and flattened shape in the reverse loading direction is modeled here by means of the concept of directional distortional hardening. The modeling of directional distortional hardening is accomplished by means of an evolving fourth-order tensor. The applicability of the model is illustrated by fitting experimental subsequent yield surfaces at finite plastic deformation. The simulation results were compared with test data for aluminum low and high work hardening alloys. In this introductory paper we will consider only the proportional loading case.

1. Introduction

The material modeling of the anisotropic plastic behavior of metallic materials represents an important issue in the numerical simulation of sheet metal forming processes. The need for an accurate description of the initial and evolving anisotropy in the material during the forming process has led to extensive research in the field. To this end, a number of papers focusing on the development of advanced yield functions to describe the onset of plastification have appeared in the open literature (see e.g. Hill, 1948, 1993; Barlat et al., 1991, 1997, 2003, 2007; Aretz, 2005; Banabic et al., 2005; Cazacu et al., 2006; Stewart and Cazacu, 2011). All of these deal with the phenomenological description of initial material anisotropy resulting from the texture induced during the rolling process of the sheet. Another essential ingredient of the phenomenological material modeling of sheet metals is the modeling of hardening. In general, one distinguishes between isotropic hardening (yield surface expansion or contraction), kinematic hardening (yield surface translation), rotational hardening (yield surface rotation without shape change) and distortional hardening (yield surface shape change). In particular, directional distortional hardening refers to a distortion of the yield surface in such a way that a region of high curvature (“nose”) develops in the direction of loading and flattening develops in the reverse direction. In this way, the concept of distortional hardening represents an instrument of incorporating evolving anisotropy in the modeling approach in the context of phenomenological constitutive modeling.

Experimental evidence concerning the distortion of the yield surface with plastic straining can be found in Naghdi et al. (1958), Phillips et al. (1975), Wu and Yeh (1991) and Boucher et al. (1995). In the works of Hill et al. (1994) and Kuwabara et al. (1998) the effect of distortional hardening in brass and steel, respectively, is observed. Iadicola et al. (2008) investigated the evolution of the yield loci during biaxial stretching of an aluminum alloy and found that rotation and elongation in the balanced biaxial direction of the yield surface takes place. An experimental study on the measurement of subsequent yield loci of laminated metal matrix composites is presented in Lissenden (2010).

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Recently, Khan et al. (2009, 2010a, have published an extensive amount of experimental data regarding the evolution of subsequent yield surfaces of aluminum alloys at finite plastic deformation. The first of these three papers (Khan et al. (2009)) deals with the experimental identification of subsequent surfaces of a very low work hardening aluminum alloy under tension, torsion and combined tension–torsion. On the other hand, the second one (Khan et al. (2010a)) presents subsequent yield surfaces for a very high work hardening aluminum alloy again under tension, torsion and combined tension–torsion. Finally, in Khan et al. (2010b) the authors present the biaxial tension results for the two aluminum alloys. The authors' results show that the initial yield surface is very close to the von Mises yield surface and the subsequent surfaces undergo translation and distortion, characterized by a nose in the loading direction and a flattening in the opposite direction. The subsequent yield surfaces for the low work hardening alloy show contraction and a negative cross-effect with finite deformation. In contrast, for the high work hardening alloy expansion and a positive cross-effect are observed.

Approaches for the modeling of distortional hardening have been presented in a number of publications, see e.g. Ortiz and Popov (1983), Voyiadjis and Foroozesh (1990), Yeh and Pan (1996), Cho and Dafalias (1996), Kowalsky et al. (1999), Francois (2001) and Aretz (2008). Ortiz and Popov (1983) developed a combined kinematic-distortional hardening model based on the J_2-plasticity theory. Francois (2001) included yield surface distortion into the classical elastoplasticity framework and thus managed to describe the egg-shape of subsequent yield surfaces. In Kowalczyk and Gambin (2004) and Plunkett et al. (2006) texture evolution is considered by coupling phenomenological yield functions with crystallographic texture models.A multiplicative description of evolving elastic and plastic anisotropy is presented in the paper of Harryson and Ristinmaa (2007). Hu (2007) described an approach to account for hardening-induced anisotropy where a direct link between directional yield stresses, r-values and yield function parameters is utilized. Vincent et al. (2004) presented a macroscopic phenomenological model taking into account yield surface distortion and applied it in the description of multiaxial ratchetting. The model is based on the framework of Kurtyka and Zyczkowski (1996) in which constitutive equations for the yield surface distortion and non-proportional multiaxial behavior have been introduced. Yeh and Lin (2006) proposed an endochronic model to describe the yield surface. They simulated the yield surface in such a way that the forward and rear parts of the yield surface are described by different ellipses which are characterized by corresponding aspect ratio functions. They obtained a good agreement between their results and experimental observations reported by Wu and Yeh (1991). A thermodynamically consistent framework of isotropic, kinematic and directional distortional hardening at small strains has been introduced in the interesting paper of Feigenbaum and Dafalias (2007). The authors made use of a fourth order tensor to describe the evolution of distortional hardening and derived all evolution equations on the basis of the fulfillment of the dissipation inequality. In Yeganeh (2007) the author introduced a finite strain rigid-plastic hardening constitutive model that incorporates distortion of the yield locus and is based on the logarithmic strain and its corotational rates. Recently, Rousselier et al. (2009) and Rousselier et al. (2010) have followed an approach for the modeling of yield locus distortion and texture evolution by utilizing a polycrystalline model where the number of crystallographic orientations is drastically reduced and a specific parameter calibration technique is used. In Rousselier et al. (2009) the authors investigate a strongly anisotropic aluminum alloy, whereas in Rousselier et al. (2010) the texture evolution of an initially isotropic fcc material is discussed.

The goal of this work is to present an approach for modeling directional distortional hardening in the regime of finite strains. In particular, we focus on developing a material model that is able to describe the experimentally observed subsequent yield loci in Khan et al. (2009) and Khan et al. (2010a). The model includes kinematic, isotropic, cross and distortional hardening and is thus able to describe yield surface translation, expansion or shrinkage, width increase perpendicular to the loading direction, and distortion. In particular, the experimentally observed increase of curvature in the loading direction together with a curvature decrease of the subsequent yield surfaces in the reverse direction can be simulated by the proposed model.

The paper is organized as follows. In Section 2 we describe the constitutive modeling of anisotropic finite multiplicative plasticity with distortional, kinematic and isotropic hardening. We discuss the evolution of the fourth-order anisotropy tensor used to describe the distortion of the yield surface with plastic deformation. In Section 3 the applicability of the material model for the prediction of subsequent yield surfaces for two aluminum alloys available in Khan et al. (2009), Khan et al. (2010a) is investigated. This is carried out by first fitting the model based on experimentally obtained subsequent yield loci in tension and then predicting the corresponding subsequent yield loci at torsion and combined tension–torsion.

2. Constitutive modeling

The material model is based on the anisotropic multiplicative elastoplasticity framework with kinematic and isotropic hardening presented in Vladimirov et al. (2010). The starting point is the classical multiplicative decomposition of the deformation gradient $\mathbf{F} = \mathbf{F}_e\mathbf{F}_p$ into elastic ($\mathbf{F}_e$) and plastic ($\mathbf{F}_p$) parts. In order to model nonlinear kinematic hardening according to the Armstrong-Frederick concept (Armstrong and Frederick (1966)), (Frederick and Armstrong (2007)) an additional multiplicative decomposition of the plastic part of the deformation gradient into elastic and inelastic parts, i.e. $\mathbf{F}_p = \mathbf{F}_{pe}\mathbf{F}_{pi}$ is introduced. This additional multiplicative split of the plastic deformation gradient can be physically motivated, see Lion (2000).

Based on the principle of material objectivity and the concept of material isomorphism the Helmholtz free energy per unit volume is given in the form:

$$\psi = \psi_e(\mathbf{C}_e) + \psi_{kin}(\mathbf{C}_{pe}) + \psi_{iso}(\kappa), \quad (1)$$