Mechanics Research Communications 69 (2015) 45–53

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Order and chaos in a three dimensional galaxy model

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ARTICLE INFO

Article history:
Received 27 November 2014
Received in revised form 29 May 2015
Accepted 9 June 2015
Available online 18 June 2015

Keywords:
Hamiltonian systems
Ordered and chaotic motion
Symplectic map of dimension four

ABSTRACT

We explore the orbital dynamics of a realistic three dimensional model describing the properties of a disk galaxy with a spherically symmetric central dense nucleus and a triaxial dark matter halo component. Regions of phase space with regular and chaotic motion are identified depending on the parameter values for triaxiality of the dark matter halo and for breaking the rotational symmetry. The four dimensional Poincaré map of the three degrees of freedom system is analyzed by a study of its restriction to various two dimensional invariant subsets of its domain.

1. Introduction

To present a global panorama of the dynamics of a system by plotting Poincaré maps has a long tradition for two degrees of freedom (2-dof) systems where these maps live on a 2 dimensional domain (for a good explanation of the idea of Poincaré maps see chapter 6 in Jackson [15]). Unfortunately things are not so easy for more degrees of freedom. Already for 3-dof the domain of the Poincaré map has dimension 4 and a graphical representation of the map is impossible. However, such 4 dimensional maps usually have the property that there are 2 dimensional invariant surfaces in the domain, i.e. surfaces with the property that initial conditions on these lower dimensional surfaces lead to trajectories which lie completely in this lower dimensional surface. And then it is easy to present graphically the restriction of the map to any one of such surfaces. Such restricted 2 dimensional maps are a great help to understand important properties of the full 4 dimensional map.

To be useful such 2 dimensional invariant surfaces must have some kind of robustness property, i.e. they must survive parameter changes such that we can follow the perturbation scenario in the restricted map. There are two types of surfaces with this property. First, the so called normally hyperbolic invariant manifolds (abbreviated NHIMs, for their general properties see Wiggins [29]), however, it seems that these objects do not play any important role in our present galaxy model. Second, lower dimensional surfaces may be invariant because of symmetry reasons, and in the present galaxy model there are various surfaces of this type which will be very useful in the following.

A further contribution to the understanding of 4 dimensional maps comes from a possible partially integrable limit case of the full system and from the corresponding possibility to build up the 4 dimensional map as stack of 2 dimensional reduced maps. Because of its importance for the present work let us give a short summary of the stack construction in a form appropriate for the case found in the dynamics of the galactic potential. Assume a Hamiltonian 3-dof system where the unperturbed case has a rotational symmetry around the z-axis and where correspondingly L, the z-component of the angular momentum, is conserved. Then the unperturbed system can be reduced to a 2-dof system which depends parametrically on the value of L. This leads to a 1 parameter family of reduced Poincaré maps each one acting on a 2 dimensional domain. From the reduced maps we obtain the non-reduced Poincaré map acting on its 4 dimensional domain in a two step process. First, we pile up the continuum of 2 dimensional reduced maps to a 3 dimensional stack where L acts as stack parameter. Second, we form the Cartesian product of this pile with a circle representing the cyclic angle canonically conjugate to the conserved quantity L. The result is a 4 dimensional construction. The full 4 dimensional Poincaré map (still for the unperturbed case) acts on the resulting 4 dimensional domain as reduced map in each invariant horizontal leaf of constant L and in addition applies a rotation to each copy of the circle. If we now add a perturbation which destroys the rotational symmetry around the z axis then the invariant foliation into the horizontal leaves is destroyed. However, if the perturbation conserves some discrete symmetry then the 4 dimensional map will have corresponding invariant 2 dimensional surfaces S also after the perturbation. These 2 dimensional surfaces do not coincide

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http://dx.doi.org/10.1016/j.mechrescom.2015.06.005
0093-6413/

with any one of the invariant 2 dimensional horizontal leaves of the unperturbed stack, they are transverse to the stack structure. Most important, the dynamics restricted to S is sensitive to the perturbation and can be exploited to obtain an understanding of the perturbation scenario for the full 3-dof system. The restricted map on S visualizes the decay of the stack structure under the perturbation. The idea works the same if the role of S is taken over by a NHIM. For all details of this approach and for examples of the stack construction see Jung et al. [16]; Gonzalez & Jung [10]; Gonzalez et al. [11]. For the analysis of a 3-dof molecular system taking advantage of this procedure see Lin et al. [21]. The present article shows that the method is equally successful for the investigation of the dynamics of a 3-dof galaxy model.

In order to explore the orbital dynamics of galaxies one should build first suitable models describing sufficiently and realistically the properties of the galaxy. Usually observations provide the necessary information on the construction of the dynamical models. A galactic model can be characterized as successful and realistic only if the derived results agree with the corresponding observational data. In most of the cases, the galaxy models are either spherical or axially symmetric. For instance, in a spherically symmetric potential all three components of the angular momentum and of course the total angular momentum are conserved. Therefore, the motion of the stars is plane and takes place in the plane perpendicular to the vector of the total angular momentum. Spherically symmetric models for galaxies were studied by Dehnen [8]; Rindler-Daller et al. [25]; Zhao [31]. In an axially symmetric potential on the other hand, only the z component of the total angular momentum is conserved. Many previous papers are devoted to the distinction between order and chaos in axially symmetric potentials (see e.g., Zotos [32,33]; Zotos & Caranicolas [36]). Furthermore, axially symmetric galaxy models were presented and examined in Cretton et al. [7].

Another interesting category of galactic potentials are the so-called composite galaxy models. In those models the potential is multi-component and each component describes a different part of the system. Composite axially symmetric galaxy models describing the motion of stars in our Galaxy were also studied by Binney [3]. In these models the gravitational potential is composed by three superposed disks: one representing the gas layer, one the thin disk and one for the thick disk. Recently in Terzić and Sprague [28], a class of realistic triaxial models for galaxies was provided. In particular, the authors extended an earlier method proposed by Terzić and Graham [27] to three-dimensional systems by replacing the radial with an ellipsoidal symmetry in the total mass density. Moreover, triaxial galaxy models were also constructed by Bailin et al [2] and Moore et al. [23].

The layout of the paper is as follows: Section 2, contains a detailed presentation of the structure and the properties of our galactic gravitational model. In Section 3, we construct Poincaré maps in order to investigate the orbital properties of the 6 dimensional phase space. The paper ends with Section 4, where the discussion and the main conclusions of our numerical analysis are presented.

2. Presentation of the galactic model

The total gravitational potential $\Phi(x, y, z)$ is three-dimensional and it consists of three components: a central, spherical component Φ_n, a flat disk Φ_d and a triaxial dark matter halo potential Φ_h.

The spherically symmetric nucleus is modeled by a Plummer potential [4]

$$\Phi_n(x, y, z) = \frac{-GM_n}{\sqrt{x^2 + y^2 + z^2 + c_n^2}}. \quad (1)$$

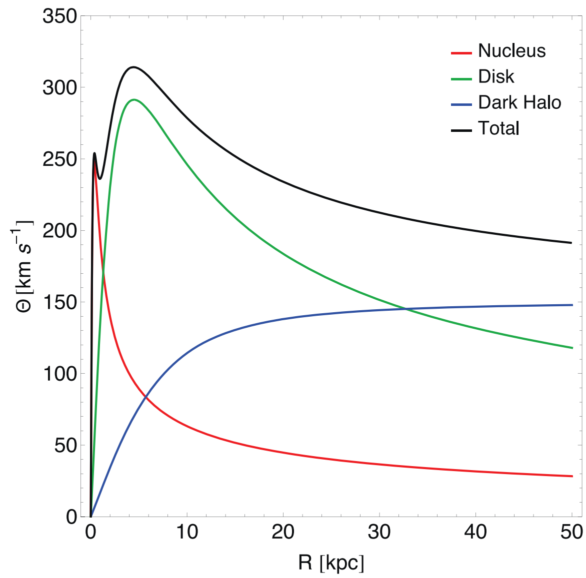

Fig. 1. A plot of the rotation curve in our galactic model. We distinguish the total circular velocity (black) and also the contributions from the central massive nucleus (red), the disk (green) and that of the dark matter halo (blue). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of the article.)

Here G is the gravitational constant, while M_n and c_n are the mass and the scale length of the nucleus, respectively. Here we must point out, that potential (1) is not intended to represent a black hole nor any other compact object, but a dense and massive bulge. Therefore, we do not include any relativistic effects.

In order to model the disk we use the Miyamoto-Nagai potential [22]

$$\Phi_d(x, y, z) = \frac{-GM_d}{\sqrt{x^2 + y^2 + \left(s + \sqrt{h^2 + z^2}\right)^2}}, \quad (2)$$

where M_d is the mass of the disk, while s and h are the horizontal and vertical scale lengths of the disk.

For the description of the properties of the dark matter halo we use the logarithmic potential

$$\Phi_h(x, y, z) = \frac{\upsilon_0^2}{2} \ln\left(x^2 + \alpha y^2 + \beta z^2 + c_h^2\right), \quad (3)$$

where α and β are the flattening parameters along the y and z axes, respectively, c_h is the scale length of the halo, while the parameter υ_0 is used for the consistency of the galactic units. The choice for the logarithmic potential was motivated for several reasons: (i) it can model a wide variety of shapes of galactic haloes by suitably choosing the parameter β. In particular, when $0.1 \leq \beta < 1$ the dark matter halo is prolate, when $\beta = 1$ is spherical, while when $1 < \beta < 2$ is oblate; (ii) it is appropriate for the description of motion in a dark matter halo as it produces a flat rotation curve at large radii (see Fig. 1); (iii) it allows for the investigation of flattened configurations of the galactic halo at low computational costs; (iv) the relatively small number of input parameters of Eq. (3) is an advantage concerning the performance and speed of the numerical model; and (v) the flattened logarithmic potential was utilized successfully in previous works to model a dark matter halo component (e.g., Helmi [12]; Ružička et al. [26]; Zotos [34]).

We use a system of galactic units, where the unit of length is 1 kpc, the unit of mass is $2.325 \times 10^7 M_\odot$ (solar masses) and the unit of time is 0.9778×10^8 yr (about 100 Myr). The velocity units is 10 km/s, the unit of angular momentum (per unit mass) is 10 km kpc s^{-1}, while G is equal to unity. The energy