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A novel method of determining the sole configuration of tensegrity structures

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ABSTRACT

A novel analysis method is presented for form-finding of tensegrity structures. The spectral decomposition of the force density matrix and the singular value decomposition of the equilibrium matrix are performed iteratively to find the feasible sets of nodal coordinates and force densities. An algorithm of determining the sole configuration of free-form tensegrities is provided by specifying an independent set of nodal coordinates, which indicates the geometrical and mechanical properties of the structures can be at least partly controlled by the proposed method. Several illustrative examples are presented to demonstrate the efficiency and robustness in finding self-equilibrium configurations of tensegrity structures.

1. Introduction

Tensegrity systems are spatial, reticulated and lightweight structures that have been known for almost half a century [1]. The makeup of these structures consists of compressed struts and tensioned cables [2]. The tensioned cables of the structure are self-stressed such that the entire system could be provided stable equilibrium before any external loads are added, including gravitational. These smart structures have a large number of potential applications, for the benefit of systems which need, for instance, a small transportation, tunable stiffness properties, active vibration damping and deployment or configuration control [3–10]. Therefore, since tensegrity systems appeared in the early 1950s, the concept of tensegrity has received significant interest among scientists and engineers throughout disciplines such as architecture [11], aerospace [12], civil engineering [13–15], robotics [16,17] to biological [18–20]. Nevertheless, a survey of current activities in research and engineering practice shows that much work has yet to be accomplished, particularly in the field of designing complex, asymmetric and free-form tensegrities.

Over the past few decades, large amounts of researches related to form-finding of tensegrity structures (including regular and irregular forms) have been performed. Schek [21] first proposed the force density method, which is widely considered the most effective and convenient form-finding procedure for tensile structures. Motro et al. [22] then applied the dynamic relaxation method to tensile structures and solved many other nonlinear problems. Following their track, Zhang et al. [23] employed the dynamic relaxation method for form-finding of nonregular tensegrities by modifying their corresponding regular ones. Zhang and Ohsaki [24] and Estrada et al. [25] proposed new numerical methods utilizing the force density formulation. Pagitz and Tur [26] advised a finite element method for form-finding of tensegrities. Most recently, Xu and Luo [27] suggested a genetic algorithm for form-finding of nonregular tensegrity structures. Tran and Lee [28] introduced a numerical method for form-finding of tensegrities with multiple states of self-stress.

Researchers have recently focused their attentions on the applications of tensegrity structures as acoustic and mechanical metamaterials [29–31], which are fresh concepts that can respond to the needs of a society in the new century. In order to better realize the optimal design and prestress tunability of tensegrity metamaterials, the sole configuration of tensegrities need to be determined. In most available form-finding methods, assumptions on either the symmetry of the structure, the element lengths or the force density coefficients must be imposed a priori. For example, (i) force density coefficients are imposed as symbolic variables [32], (ii) a global symmetry is assumed in a group theory so as to simplify form-finding procedure [33], (iii) some of the member lengths are prespecified in a dynamic relaxation process and non-linear programming [22,31]. However, this kind of information may not always be available or easy to

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define beforehand. The computation of complex and/or asymmetric tensegrity structures with these procedures therefore remains difficult.

The present paper is an extension of Estrada's work [25] and is aimed at form-finding of free-form tensegrity structures in which both Classes 1 (where bars do not touch) and 2 (where bars do connect to each other at a pivot) of tensegrity structures are investigated. Tensegrities satisfying either stability (i.e., the tangent stiffness matrix is positive definite) or super stability (i.e., the geometrical stiffness matrix is positive definite) can be achieved by present form-finding process in a few of remarkable iterations. Hence, compared to other available methods only dealing with super stable tensegrity structures [23,25] that are more restrictive in the real mechanical structures, the proposed form-finding is more efficient and versatile. The dimension of the structure, the connectivity between the nodes and the type of each member are the only required information in this numerical form-finding procedure. The spectral decomposition of force density matrix and the singular value decomposition of the equilibrium matrix are performed iteratively to find the feasible sets of nodal coordinates and force densities which satisfy the necessary minimum rank deficiency conditions of force density and equilibrium matrices, respectively. In other words, any assumption about initial nodal coordinates or element lengths, material properties, structural symmetry and the positive semi-definite condition of the force density matrix is not necessary in the proposed form-finding procedure, which is considered as the advantage of this method compared to the available ones. An approach of defining a sole configuration of free-form tensegrity structures by specifying an independent set of nodal coordinates is provided, which indicates that the geometrical and mechanical properties of the structure can be at least partly handled by the proposed method. The evaluation of the eigenvalues of tangent stiffness matrix is also included for checking the stability of the tensegrity structures.

2. Force density method

2.1. Fundamental assumptions

It is acknowledged known that the form-finding of a tensegrity structure is very similar to that of a cable net, because they use almost the same fundamental assumptions listed as follows except the last two ones that are suitable only for tensegrity structures:

- The topology (connectivity between the nodes and members) of the structure is known, and the geometrical configuration of the structure can be described in terms of nodal coordinates only.
- Members are connected by pin joints.
- No external load is applied and the self-weight of the structure is neglected.
- Both local and global bucking are not considered during the form-finding procedure.
- The structure is free-standing without any support; i.e. there are no dissipative forces acting on the system.

2.2. Self-equilibrium equations for free-standing tensegrity structures

For a d-dimensional ($d = 2$ or 3) tensegrity structure with b members, n free nodes and n_f fixed nodes, its topology can be described by a connectivity matrix $\mathbf{C}_S \in \boldsymbol{R}^{b\times(n+n_f)}$ as defined in [2]. If member k connects nodes i and j ($i<j$), then the ith and jth elements of the kth row of $\mathbf{C}_S$ are set to 1 and -1, respectively, as follows

$$\mathbf{C}_{S(k,p)} = \begin{cases} 1 & \text{for} \quad p = i \\ -1 & \text{for} \quad p = j \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The fixed nodes are preceded by the free nodes in the numbering sequence, then $\mathbf{C}_S$ can be divided into two parts as

$$\mathbf{C}_S = \begin{bmatrix} \mathbf{C} & \mathbf{C}_f \end{bmatrix} \tag{2}$$

where $\mathbf{C} \in \boldsymbol{R}^{b\times n}$ and $\mathbf{C}_f \in \boldsymbol{R}^{b\times n_f}$ describe the connectivities of the members to the free and fixed nodes, respectively.

Let $\mathbf{x}, \mathbf{y}, \mathbf{z} (\in \boldsymbol{R}^n)$ and $\mathbf{x}_f, \mathbf{y}_f, \mathbf{z}_f (\in \boldsymbol{R}^{n_f})$ denote the nodal coordinate vectors of the free and fixed nodes, respectively, in x-, y- and z-directions. The force density coefficients vector is denoted by $\mathbf{q} = \left\{q_1, q_2, \ldots, q_b\right\}^T \in \boldsymbol{R}^b$, in which each component of this vector is the force f_k to length l_k ratio $q_k = f_k/l_k$ ($k = 1, 2, \ldots, b$) known as force density or self-stressed coefficient in [34]. The force density matrix $\mathbf{Q} \in \boldsymbol{R}^{b\times b}$ is given as

$$\mathbf{Q} = \mathrm{diag}(\mathbf{q}) \tag{3}$$

The equilibrium equations of the free nodes in each direction of a general pin-jointed structure can be written as follows [21]

$$\mathbf{C}^T\mathbf{QCx} + \mathbf{C}^T\mathbf{QC}_f\mathbf{x}_f = \mathbf{p}_x \tag{4.1}$$

$$\mathbf{C}^T\mathbf{QCy} + \mathbf{C}^T\mathbf{QC}_f\mathbf{y}_f = \mathbf{p}_y \tag{4.2}$$

$$\mathbf{C}^T\mathbf{QCz} + \mathbf{C}^T\mathbf{QC}_f\mathbf{z}_f = \mathbf{p}_z \tag{4.3}$$

where $\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z (\in \boldsymbol{R}^n)$ are the vectors of external loads applied at the free nodes in x-, y- and z- directions, respectively.

For simplicity, matrices $\mathbf{E} \in \boldsymbol{R}^{n\times n}$ and $\mathbf{E}_f \in \boldsymbol{R}^{n\times n_f}$ are defined as

$$\mathbf{E} = \mathbf{C}^T\mathbf{QC} \tag{5.1}$$

$$\mathbf{E}_f = \mathbf{C}^T\mathbf{QC}_f \tag{5.2}$$

Note that $\mathbf{E}$ and $\mathbf{E}_f$ are constant when the force density matrix $\mathbf{Q}$ is given.

From Eq. (5.1), $\mathbf{E}$ is always square, symmetric and singular with a nullity of at least one since the sum of all its components in any row or column is zero for any tensegrity structure [2].

When external load and self-weight are ignored, the tensegrity systems does not require any fixed nodes. Its geometry can be defined by the relative positions of the nodes. Thus, the system can be considered as a free-standing rigid-body in space [34]. Eqs. (4.1)–(5.2) becomes

$$\mathbf{Ex} = 0 \tag{6.1}$$

$$\mathbf{Ey} = 0 \tag{6.2}$$

$$\mathbf{Ez} = 0 \tag{6.3}$$

For simplicity, Eqs. (6.1)–(6.3) can be reorganized as

$$\mathbf{E}[\,\mathbf{x} \quad \mathbf{y} \quad \mathbf{z}\,] = \mathbf{C}^T\mathbf{QC}[\,\mathbf{x} \quad \mathbf{y} \quad \mathbf{z}\,] = [\,\mathbf{0} \quad \mathbf{0} \quad \mathbf{0}\,] \tag{7}$$

where $[\mathbf{x}\ \mathbf{y}\ \mathbf{z}] (\in \boldsymbol{R}^{n\times d})$ is a matrix of nodal coordinates for a d-dimensional ($d = 2$ or 3) tensegrity structure.

As can be seen, Eq. (7) presents the relation between the force densities and the nodal coordinates.

For simplicity, matrices $\mathbf{D}_x, \mathbf{D}_y, \mathbf{D}_z (\in \boldsymbol{R}^{b\times b})$ are defined as

$$\mathbf{D}_x = \mathrm{diag}(\mathbf{Cx}) \tag{8.1}$$

$$\mathbf{D}_y = \mathrm{diag}(\mathbf{Cy}) \tag{8.2}$$

$$\mathbf{D}_z = \mathrm{diag}(\mathbf{Cz}) \tag{8.3}$$