Mechanics Research Communications 69 (2015) 91–96

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Symmetric-Galerkin boundary element analysis of the dynamic *T*-stress for the interaction of a crack with an auxetic inclusion

K. Kwon, A.-V. Phan*

Department of Mechanical Engineering, University of South Alabama, Mobile, AL 36688, USA

ARTICLE INFO

Article history:
Received 22 November 2014
Received in revised form 23 June 2015
Accepted 29 June 2015
Available online 7 July 2015

Keywords:
Symmetric-Galerkin boundary element method
Laplace domain
Dynamic stress intensity factors
Dynamic *T*-stress
Auxetic materials

ABSTRACT

In this work, the symmetric-Galerkin boundary element method (SGBEM) for 2-D elastodynamics in the Laplace-space frequency domain (Laplace domain) is employed to study the dynamic stress intensity factors (DSIFs) and the dynamic *T*-stress (DTS) during the interaction between a crack and an auxetic inclusion under impact loading conditions. It is found that, while the auxeticity has virtually no effect on the DSIFs, its influence on the DTS is noticeable. This finding is particularly important as it implies the imperative need of fracture criteria based on both the DSIFs and DTS for predicting crack growth in composite materials with auxetic phases.

1. Introduction

Materials with negative Poisson's ratio(s) are referred to as auxetic. They are known to possess various better physical characteristics than those of non-auxetic materials. Auxetic foams were first made by Lakes [1] and quickly became of interest to the science community. A comprehensive review on current understanding of both natural and man-made auxetic materials can be found in [2]. Potential applications of auxetic composites are wide ranging, such as wear resistant machine components, gaskets, piezoelectric sensors in hydrophones and ultrasonic devices, efficient noise and vibration absorbers [3], molecular sieves [4], expandable blast-proof curtains [5], etc.

Various models have been developed to explain auxetic behaviors (*e.g.*, [6]), to study the effects of auxeticity on material properties (*e.g.*, [7]). Choi and Lakes [8] found that the fracture toughness of auxetic copper foams is enhanced as permanent volumetric compression ratio is increased. Very recently, a numerical study [9] has shown a dramatic difference in quasi-static crack growth behavior near auxetic particles as compared to non-auxetic ones. Especially, it is found that auxetic particles could be employed to attract/pin a growing matrix crack. In spite of these interesting discoveries, to the best of our knowledge, no work on the dynamic fracture behavior of composites with auxetic phases, such as auxetic particulate-filled composites, has been reported in the literature. In an effort to explore this research area, we introduced herein a numerical investigation of the two most important fracture parameters, namely the dynamic stress intensity factor (DSIF) and the dynamic *T*-stress (DTS), during the interaction between a crack and an auxetic inclusion under impact loading conditions. This investigation was carried out using the symmetric Galerkin boundary element method (SGBEM) in the Laplace domain as this technique has shown to be accurate and effective in computing the aforementioned fracture parameters [10].

The objective of this work is to investigate the influence of auxeticity on the fracture behavior in auxetic particulate-filled composites. To this end, we compared the DSIF and DTS during the dynamic interaction between a crack and an auxetic inclusion with those using a non-auxetic inclusion. Three different situations of crack-inclusion interaction under impact loading conditions were considered and they all led to the same finding.

2. Symmetric boundary integral formulations for elastodynamics in the Laplace domain

First, consider the body in Fig. 1 without the crack. In the Laplace domain, the BIE for a source point P interior to a domain is given by

$$\overline{\mathcal{U}}(P,s) \equiv \overline{u}_k(P,s) - \int_\Gamma [\overline{U}_{kj}(P,Q,s)\overline{t}_j(Q,s) - \overline{T}_{kj}(P,Q,s)\overline{u}_j(Q,s)]\mathrm{d}Q = 0 \tag{1}$$

* Corresponding author. Tel.: +1 251 460 7453; fax: +1 251 460 6549.
E-mail address: vphan@southalabama.edu (A.-V. Phan).

http://dx.doi.org/10.1016/j.mechrescom.2015.06.015
0093-6413/

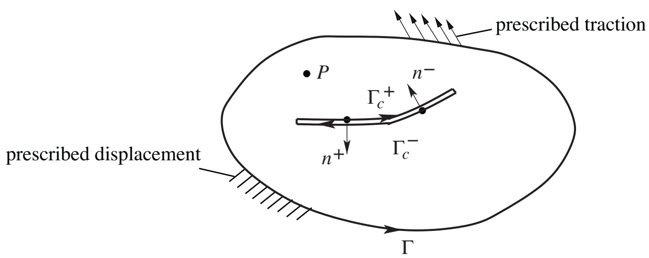

Fig. 1. A 2-D body containing a crack.

where Q is a filed point, s is a Laplace parameter, $\bar{u}_j$ and $\bar{t}_j$ denote the transformed displacement and traction vectors, respectively, and Γ is the boundary of the 2-D domain under consideration.

For P off the boundary, the kernel functions are not singular and it is permissible to differentiate Eq. (1) with respect to P, yielding the transformed displacement gradients. Substitution of these gradients into Hooke's law and then Cauchy's relation results in the equation for surface traction,

$$\bar{\mathcal{T}}(P,s) \equiv \bar{t}_k(P,s) - n_\ell(P)\int_\Gamma [\bar{D}_{kj\ell}(P,Q,s)\bar{t}_j(Q,s) - \bar{S}_{kj\ell}(P,Q,s)\bar{u}_j(Q,s)]\mathrm{d}Q = 0 \quad (2)$$

It can be shown that the limits of the integrals in Eqs. (1) and (2) as P approaches the boundary exist. From now on, for $P \in \Gamma$, the BIE is understood in this limiting sense. Expressions for the elastodynamic kernel tensors $\bar{U}_{kj}$, $\bar{T}_{kj}$, $\bar{D}_{kj\ell}$ and $\bar{S}_{kj\ell}$ in Eqs. (1) and (2) can be found in, *e.g.*, [11].

The traction Eq. (2) is essential for treating crack geometries, and in the symmetric-Galerkin approach it is this equation that is employed on the crack surface.

The Galerkin boundary integral formulation is obtained by taking the shape functions ψ_m employed in approximating the boundary tractions and displacements as weighting functions for Eqs. (1) and (2). For SGBEM, the displacement BIE (1) is employed on the boundary part Γ_u where displacements are specified, while the traction BIE (2) is used on the boundary part Γ_t where tractions are prescribed,

$$\int_{\Gamma_u} \psi_m(P)\bar{\mathcal{U}}(P,s)\,\mathrm{d}P = 0 \quad (3)$$

$$\int_{\Gamma_t} \psi_m(P)\bar{\mathcal{T}}(P,s)\,\mathrm{d}P = 0 \quad (4)$$

As the name implies, this arrangement results in a symmetric coefficient matrix. These formulas remain the same for fracture analysis, with the proviso that the unknowns on the crack are now the transformed displacement jump $\Delta\bar{u}_k$, and thus only one crack face is required to be discretized. In this work, a numerical implementation of Eqs. (3) and (4) is carried out with the standard quadratic element. Employing the parameter space $\xi \in [0, 1]$, and defining $\xi_1 = 0$, $\xi_2 = 1/2$ and $\xi_3 = 1$, the quadratic shape functions are defined by $\psi_\ell(\xi_m) = \delta_{\ell m}$ and hence

$$\psi_1(\xi) = (1-\xi)(1-2\xi) \quad (5)$$

$$\psi_2(\xi) = 4\xi(1-\xi) \quad (6)$$

$$\psi_3(\xi) = \xi(2\xi-1) \quad (7)$$

One of the advantages of the frequency-domain analysis is that these formulations have a similar form as those in elastostatics. The reader is referred to, for example, Ref. [12] for more details on the SGBEM.

The above SGBEM formulations need to be extended to deal with multiregion problems such as those involving the interaction between a crack and auxetic inclusions considered in this work. The multiregion technique is based on the assumption of a perfect bonding between inclusions and the matrix which results in the displacement continuity and traction equilibrium conditions across the interfaces. This technique should be appicable to both non-auxetic and auxetic inclusions. More details of the multiregion SGBEM can be found in, *e.g.*, [13].

3. Fracture analysis by the SGBEM for Laplace-space elastodynamics

Frequency domain analysis of cracks, such as the analysis of the DSIFs and DTS, using the SGBEM in the Laplace space was introduced in [10]. In this section, the formulations for this type of analysis are briefly reviewed.

3.1. Formulations

If a crack of boundary Γ_c is added to the domain of boundary $\Gamma = \Gamma_u \cup \Gamma_t$ considered in the previous section, the new total boundary becomes $\overset{*}{\Gamma} = \Gamma \cup \Gamma_c$ (see Fig. 1). The crack is composed of two symmetrically loaded surfaces Γ_c^+ and Γ_c^- which are initially coincident. Let $\Gamma_t^* = \Gamma_t + \Gamma_c^+$. In this case, the displacement and traction BIEs are written as

$$\overset{*}{\mathcal{U}}(P,s) \equiv \bar{\mathcal{U}}(P,s) + \int_{\Gamma_c^+} \bar{T}_{kj}(P,Q,s)\,\Delta\bar{u}_j(Q,s)\,\mathrm{d}Q = 0 \quad (8)$$

$$\overset{*}{\mathcal{T}}(P,s) \equiv \bar{\mathcal{T}}(P,s) + n_\ell^+(P)\int_{\Gamma_c^+} \bar{S}_{kj\ell}(P,Q,s)\,\Delta\bar{u}_j(Q,s)\,\mathrm{d}Q = 0 \quad (9)$$

where n_ℓ^+ is the outward normal vector to Γ_c^+, and as the transformed displacement jump vector $\Delta\bar{u}_j$ across the crack surfaces is used as the unknown on the crack, only one crack surface, *e.g.*, Γ_c^+, needs to be discretized. It is well known that the traction BIE (9) is essential for treating crack geometries.

The use of $\Delta\bar{u}_j$ as the unknown on the crack as mentioned above is needed for obtaining a symmetric coefficient matrix. The symmetric-Galerkin formulation is given by

$$\int_{\Gamma_u} \psi_m(P)\overset{*}{\mathcal{U}}(P,s)\,\mathrm{d}P = 0 \quad (10)$$

$$\int_{\overset{*}{\Gamma}_t} \psi_m(P)\overset{*}{\mathcal{T}}(P,s)\,\mathrm{d}P = 0 \quad (11)$$

3.2. Dynamic stress intensity factors

For stationary cracks (as those considered in this work), the DSIFs, and thereby their transforms can be determined from the asymptotic expansion for the displacement field in the vicinity of a crack tip as follows:

$$\begin{aligned} \bar{K}_{\mathrm{I}}(s) &= \frac{\mu_c}{4(1-\nu)}\lim_{r\to 0}\sqrt{\frac{2\pi}{r}}\,\Delta\bar{u}_n(s) \\ \bar{K}_{\mathrm{II}}(s) &= \frac{\mu_c}{4(1-\nu)}\lim_{r\to 0}\sqrt{\frac{2\pi}{r}}\,\Delta\bar{u}_t(s) \end{aligned} \quad (12)$$

where $\Delta\bar{u}_n$ and $\Delta\bar{u}_t$ are, respectively, the normal and tangential components of the transformed crack displacement jump vector, and r is the distance to the crack tip.

In both finite and boundary element modeling of discrete cracks, the standard approach consists of incorporating the quarter-point (QP) element [14,15] to improve the accuracy of the SIF calculations (*e.g.*, [16,17]). However, as discussed in [18], the standard QP