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Large amplitude free vibration of symmetrically laminated magneto-electro-elastic rectangular plates on Pasternak type foundation

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ABSTRACT

Nonlinear free vibration of symmetrically laminated magneto-electro-elastic rectangular plate resting on an elastic foundation is studied analytically. The plate is considered to be simply supported on all edges. It is also assumed that the magneto-electro-elastic body is poled along the z direction and subjected to electric and magnetic potentials between the upper and lower surfaces. To model the motion of the plate, the first order shear deformation theory along with the Gauss's equations for electrostatics and magnetostatics are used. Then equations of motion are reduced to a single nonlinear ordinary differential equation which is solved analytically by multiple scales method. The results are compared with the published results and good agreement is found. Some numerical examples are presented to investigate the effects of several parameters on the linear and nonlinear behavior of these plates.

1. Introduction

In magneto-electro-elastic (MEE) composite materials a coupling between mechanical, electric and magnetic fields results in the ability to convert energy among these three energy forms. These materials have direct application in sensors and actuators, damping and control of vibrations in structures.

Pan [13] studied multilayered MEE plates analytically for the first time. Heyliger [14] derived analytical solutions for free vibrations of these smart plates. Ramirez et al. [15] presented an approximate solution for the free vibration problem of MEE plates. Milazzo [8,9] presented single-layer approaches to static and free vibration analysis of MEE laminated plates. Recently, Chen et al. [3] studied the free vibration of multilayered MEE plates under combined clamped/free boundary conditions. Li and Zhang [6] studied the free vibration of a MEE plate resting on a Pasternak foundation by using the Mindlin theory.

In the nonlinear field, few studies on the nonlinear behavior of MEE plates are available. Xue et al. [19] studied the large deflection of a rectangular MEE thin plate for the first time based on the classical plate theory. Sladek et al. [17] used a meshless local Petrov–Galerkin (MLPG) method to study the large deflection of MEE thick plates. Milazzo [10] and Alaimo et al. [1] presented a shear deformable model and an original finite element formulation, respectively, for the large deflection analysis of magneto-electro-elastic laminated plates.

To the best of the authors' knowledge, the effects of electric and magnetic potentials on the nonlinear vibration of MEE plates have not been investigated. So, this study is done to fill the gap in this area. The purpose of this paper is to study the linear and nonlinear free vibration of multilayered MEE plates with simply supported boundary conditions. To this end, the first order shear deformation theory (FSDT) along with Gauss's equations for electrostatics and magnetostatics and Galerkin method are used to obtain the governing equations of these plates. The obtained equation is solved analytically and some numerical examples are given to investigate the effects of several parameters on the nonlinear behavior of these smart plates.

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2. Governing equations

The strain–displacement relations based on the FSDT are [16]:

$$\varepsilon_x=\varepsilon_x^0+z\varepsilon_x^1=\left(u_{0,x}+\frac{1}{2}w_{0,x}^2\right)+z\theta_{x,x},\ \varepsilon_y=\varepsilon_y^0+z\varepsilon_y^1=\left(v_{0,y}++\frac{1}{2}w_{0,y}^2\right)+z\theta_{y,y},\ \gamma_{xy}=\gamma_{xy}^0$$
$$+z\gamma_{xy}^1=(u_{0,y}+v_{0,x}+w_{0,x}w_{0,y})+z(\theta_{x,y}+\theta_{y,x}),\ \gamma_{yz}=w_{0,y}+\theta_y,\ \gamma_{xz}=w_{0,x}+\theta_x \tag{1}$$

where u_0, v_0, and w_0 are the displacements of the mid-surface along x, y, and z directions, respectively, and θ_x and θ_y are the rotations of a transverse normal about the y and x directions, respectively.

For the kth layer of a MEE material, the constitutive relations can be written as [6]:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{Bmatrix}^{(k)}=\begin{bmatrix}C_{11}&C_{12}&0&0&0\\ C_{12}&C_{22}&0&0&0\\ 0&0&C_{44}&0&0\\ 0&0&0&C_{55}&0\\ 0&0&0&0&C_{66}\end{bmatrix}^{(k)}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}^{(k)}-\begin{bmatrix}0&0&e_{31}\\ 0&0&e_{32}\\ 0&e_{24}&0\\ e_{15}&0&0\\ 0&0&0\end{bmatrix}^{(k)}\begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix}^{(k)}-\begin{bmatrix}0&0&q_{31}\\ 0&0&q_{32}\\ 0&q_{24}&0\\ q_{15}&0&0\\ 0&0&0\end{bmatrix}^{(k)}\begin{Bmatrix}H_x\\ H_y\\ H_z\end{Bmatrix}^{(k)}, \tag{2}$$

$$\begin{Bmatrix}D_x\\ D_y\\ D_z\end{Bmatrix}^{(k)}=\begin{bmatrix}0&0&0&e_{15}&0\\ 0&0&e_{24}&0&0\\ e_{31}&e_{32}&0&0&0\end{bmatrix}^{(k)}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}^{(k)}+\begin{bmatrix}\eta_{11}&0&0\\ 0&\eta_{22}&0\\ 0&0&\eta_{33}\end{bmatrix}^{(k)}\begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix}^{(k)}+\begin{bmatrix}d_{11}&0&0\\ 0&d_{22}&0\\ 0&0&d_{33}\end{bmatrix}^{(k)}\begin{Bmatrix}H_x\\ H_y\\ H_z\end{Bmatrix}^{(k)}, \tag{3}$$

$$\begin{Bmatrix}B_x\\ B_y\\ B_z\end{Bmatrix}^{(k)}=\begin{bmatrix}0&0&0&q_{15}&0\\ 0&0&q_{24}&0&0\\ q_{31}&q_{32}&0&0&0\end{bmatrix}^{(k)}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}^{(k)}+\begin{bmatrix}d_{11}&0&0\\ 0&d_{22}&0\\ 0&0&d_{33}\end{bmatrix}^{(k)}\begin{Bmatrix}E_x\\ E_y\\ E_z\end{Bmatrix}^{(k)}+\begin{bmatrix}\mu_{11}&0&0\\ 0&\mu_{22}&0\\ 0&0&\mu_{33}\end{bmatrix}^{(k)}\begin{Bmatrix}H_x\\ H_y\\ H_z\end{Bmatrix}^{(k)}, \tag{4}$$

where $\{\sigma_x\sigma_y\sigma_{xz}\sigma_{yz}\sigma_{xy}\}^T$ and $\{\varepsilon_x\varepsilon_y\gamma_{yz}\gamma_{xz}\gamma_{xy}\}^T$ are stress and strain vectors, respectively; $\{D_xD_yD_z\}^T$ and $\{B_xB_yB_z\}^T$ are the electric displacement and magnetic flux density vectors, respectively; $\{E_xE_yE_z\}^T$ and $\{H_xH_yH_z\}^T$ are electric field and magnetic field intensity vectors, respectively; $[C_{ij}]$, $[\eta_{ij}]$ and $[\mu_{ij}]$ are the elastic, dielectric and magnetic permeability coefficient matrices, respectively; and $[e_{ij}]$, $[q_{ij}]$ and $[d_{ij}]$ are the piezoelectric, piezomagnetic and magnetoelectric coefficient matrices, respectively.

The governing equations of motion of a MEE plate are [16,5,10]:

$$N_{x,x}+N_{xy,y}=0 \tag{5}$$

$$N_{xy,x}+N_{y,y}=0 \tag{6}$$

$$Q_{x,x}+Q_{y,y}+(N_xw_{0,x}+N_{xy}w_{0,y})_{,x}+(N_{xy}w_{0,x}+N_yw_{0,y})_{,y}-k_ww_0+k_s\nabla^2w_0=I_0w_{0,tt} \tag{7}$$

$$M_{x,x}+M_{xy,y}-Q_x=0 \tag{8}$$

$$M_{xy,x}+M_{y,y}-Q_y=0 \tag{9}$$

$$D_{x,x}^{(k)}+D_{y,y}^{(k)}+D_{z,z}^{(k)}=0 \tag{10}$$

$$B_{x,x}^{(k)}+B_{y,y}^{(k)}+B_{z,z}^{(k)}=0 \tag{11}$$

where Eqs. (5)–(9) are equations of motion of a plate based on the FSDT, and Eqs. (10) and (11) are Gauss's equations in electrostatics and magnetostatics. k_w and k_s are spring and shear coefficients of the Pasternak foundation, respectively. N_x, N_y, and N_{xy} are the in-plane force resultants, Q_x and Q_y are the transverse force resultants, M_x, M_y, and M_{xy} are the moments resultants and I_0 is the mass moments of inertia and are obtained by:

$$\{N_xN_yN_{xy}\}^T=\int_{\frac{-h}{2}}^{\frac{h}{2}}\{\sigma_x\sigma_y\sigma_{xy}\}^Tdz,\quad \{M_xM_yM_{xy}\}^T=\int_{\frac{-h}{2}}^{\frac{h}{2}}\{\sigma_x\sigma_y\sigma_{xy}\}^Tzdz,\quad \{Q_xQ_y\}^T=K\int_{\frac{-h}{2}}^{\frac{h}{2}}\{\sigma_{xz}\sigma_{yz}\}^Tdz,\quad I_0=\int_{\frac{-h}{2}}^{\frac{h}{2}}\rho_0dz \tag{12}$$

where K is shear correction factor and ρ_0 is the density of the material of plate.The components of the electric and magnetic fields are assumed to be [1]:

$$E_x^{(k)}=E_y^{(k)}=H_x^{(k)}=H_y^{(k)}=0;\quad E_z^{(k)}=-\phi_{,z}^{(k)},\quad H_z^{(k)}=-\psi_{,z}^{(k)} \tag{13}$$