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Chaotic vibrations of beams on nonlinear elastic foundations subjected to reciprocating loads

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ABSTRACT

Chaotic vibration of beams resting on a foundation with nonlinear stiffness is investigated in this paper. Cosine–cosine function is employed in modeling of the reciprocating load. The equation of motion is derived and solved to obtain corresponding Poincaré section in phase–space. Lyapunov exponent as a criterion for chaos indication is obtained. Dynamic behavior of the beam is examined in resonance condition. Homoclinic orbits are captured and their corresponding Melnikov's functions are established. A parametric study is then carried out and effects of linear and nonlinear parameters on the chaotic behavior of the system are studied.

1. Introduction

Chaotic behavior of nonlinear dynamical systems has received very much interest both from scientists and engineers. This focus generally is raised owing to losing predictability and consequent controllability of the dynamic systems. Among these dynamical systems, continuous types such as beams, plates, and shells generally have wider range of applications and subsequent research load.

Chaotic vibration of an Euler–Bernoulli beam subjected to simultaneous transversal and impact loading was studied by Awrejcewicz et al. [1]. Vestroni [2] analyzed the dynamic behavior of a simply supported beam subjected to an axial transport of mass. He obtained a homoclinic orbit in a high dimensional phase space and studied its stability and collapse. Chaotic motion of viscoelastic beams with geometric and physical nonlinearities has been studied by Chen et al. [3]. They employed phase plane trajectory, power spectrum and Lyapunov exponents to investigate chaotic behavior of the beam. Chaotic motion of axial compressed nonlinear elastic beam subjected to transverse load has been studied by Zhang et al. [4]. Using the Melnikov's theorem they obtained threshold values of the load parameters in which, chaos may occur in the system. Liu et al. [5] analyzed symmetrical, asymmetrical, and chaotic responses of elastic–plastic beams under symmetrical impulsive loading. Phase plane trajectories, Poincaré maps and power spectral density diagrams were used to identify both symmetrical and asymmetrical chaotic vibrations. Chaotic dynamics of the softening Duffing oscillator with multi frequency external periodic forces have been studied by Lou et al. [6]. They used Melnikov's approach to obtain heteroclinic responses and to find necessary and sufficient conditions for chaos appearance.

Calio and Elishakoff [7] presented closed-form trigonometric solutions for inhomogeneous beam-columns on elastic foundations. Sunil [8] investigated the dynamic response of a Timoshenko beam under repeated pulse loading. Effects of the beam tip rub forces on the dynamic stability of a spinning blade with intermittent rub were studied in that reference. Chaotic response of large-amplitude vibration of beams has been analyzed by Han and Zheng [9]. They used Melnikov's function method to obtain the chaotic critical conditions for the single mode model. Yang and Chen [10] employed Poincaré approach and investigated bifurcation and chaos in an axially accelerating viscoelastic beam applying Kelvin–Voigt model. Melnikov's method was used to determine safe and unsafe zooms in the force-parameter space by Santee et al. [11]. The Poincaré map, the Fourier spectra, the maximum Lyapunov exponents and the principal component analysis were used by Nagai et al. [12] to identify chaotic behavior of thin-walled, post-buckled beams subjected to periodic lateral accelerations. Battelli et al. [13] studied a PDE modeling of a compressed beam with small friction and subjected to a periodic forcing of small amplitude. Effects of the temperature, the axial load and the environmental damping on the chaotic behavior of shape memory

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alloy beams were investigated by Qingquan [14]. Three different methods including Galerkin procedure, harmonic balance approach and Runge–Kutta–Gill method have been employed by Yanagisawa et al. [15] to study the chaotic vibrations of a clamped-supported beam with a concentrated mass subjected to lateral periodic acceleration. Ghayesh and Balar [16] studied an axially moving viscoelastic Rayleigh beam with cubic nonlinearity. They investigated effects of different parameters on the vibrational behavior, stability and bifurcation points of the system through a parametric study. They [17] also investigated the stability characteristics of an axially accelerating string supported by an elastic foundation.

Akour [18] analyzed simply supported nonlinear beam resting on linear elastic foundation and subjected to harmonic loading. Using the method of multiple scales, Amer and Hegazy [19] examined the nonlinear behavior of a string-beam coupled system subjected to parametric excitation. They carried out a numerical simulation to study the steady state response, stable solutions and chaotic motions. Onozato et al. [20] analyzed chaotic vibrations of a post-buckled L-shaped beam with an axial constraint. Chaotic dynamics of flexible Euler–Bernoulli beams has been studied by Awrejcewicz et al. [21]. They analyzed time histories, phase and modal portraits, autocorrelation functions, the Poincaré and pseudo-Poincaré maps, signs of the first four Lyapunov exponents, as well as the compression factor of the phase volume of an attractor. Bifurcation and chaotic behavior of an axially accelerating viscoelastic beam in supercritical regime has been studied by Ding et al. [22]. They used Galerkin truncation as well as the differential and integral quadrature method to investigate the nonlinear dynamic behavior of the system.

Nonlinear flexural waves and chaotic behavior in Timoshenko beams was studied by Zhang and Liu using the method of Jacobi elliptic function expansion [23]. Younesian and Norouzi [24] analyzed forced vibration analysis of spinning disks subjected to transverse forces. They used Galerkin's approach in order to solve the equation of motion of the rotating disk. They reported that the increasing in the spinning speed leads to increase in natural frequencies of the spinning disk. Frequency analysis of the nonlinear viscoelastic plates subjected to subsonic flow and external excitation was studied by Younesian and Norouzi [25]. They used Bernoulli's principal to model the pressure acting on the plate surface. Galerkin's approach was used in their study in order to transform the partial differential equation of motion of the plate into the ordinary differential equation. They found the critical speed of the flow in which the plate can exhibit unstable behaviors. Awrejcewicz and Pyryev [26] used Melnikov's function in order to predict the chaos in the Duffing-type system with friction. They showed that the obtained corresponding Melnikov's function can be simplified in some cases to yield analytical conditions for chaos prediction. Their results have been verified by numerical analysis. Prediction of chaos in the rotated Froude pendulum has been performed analytically by Awrejcewicz and Holicke [27]. They studied a rotated Froude pendulum with coloumb-type friction, viscous damping and external harmonic excitation to analyze chaotic dynamics in such a system. Melnikov's approach has been used by them to predict chaos behavior of the system and the analytical results have been confirmed by numerical simulations. Andrianov et al. [28] developed modified Muravskii model for elastic foundation. They established a frequency equation for the vibration of an engine seating and equation for the pressure under the bottom of the engine.

Apart from various types of structures under machining and finishing process, nowadays many other nano and microstructures are found to be influenced by reciprocating loading. In the present study we investigate potential chaotic oscillations in beams rested on a nonlinear foundation and subjected to a reciprocating type of loading. Corresponding equation of motion are solved using the Galerkin's method. Poincaré section and Lyapunov exponent are taken into account. Corresponding Melnikov's function is developed to determine analytical conditions of chaotic motion. A parametric study is carried out over a range of input parameters and effect of those on the dynamic characteristic behavior of the system is investigated.

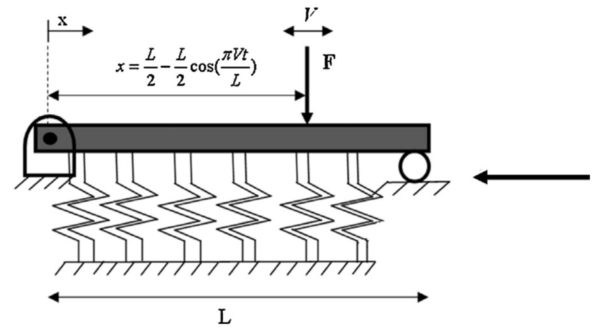

Fig. 1. The beam on the nonlinear elastic foundation subjected to reciprocating loading.

2. Governing equation of motion

Fig. 1 shows the beam rested on a nonlinear elastic foundation which consists of a pair of parallel linear and nonlinear springs. A concentrated load F goes toward from the origin ($x=0$) and comes backward to that point. The beam has simply supported boundary conditions and has been expected to follow Euler–Bernoulli assumptions. Moreover, nonlinear spring has been assumed to behave as the rule $F=Kw^3$. Equation of motion for the beam can be expressed as [29–31]

$$EI\frac{\partial^4 w}{\partial x^4}+N\frac{\partial^2 w}{\partial x^2}+\rho A\ddot{w}+K_1 w+K_2 w^3=f(x,t) \tag{1}$$

In which E denotes the modulus of elasticity, I is moment of inertia, ρ is mass density, A is cross section area, L stands for the beam length and N denotes the compressive axial force. Furthermore, K_1 and K_2 are the linear and nonlinear stiffness respectively.At the first one can introduce the dimensionless parameters as

$$w^*=\frac{w}{L}\quad x^*=\frac{x}{L}\quad t^*=\frac{tV}{L}\quad \omega^*=\frac{\omega L}{V}\quad \Omega^*=\frac{\Omega L}{V}\quad N^*=\frac{N}{\rho AV^2}$$
$$K_0^*=\frac{EI}{\rho AL^2V^2}\quad K_1^*=\frac{K_1L^2}{\rho AV^2}\quad K_2^*=\frac{K_2L^4}{\rho AV^2}\quad f^*(x^*,t^*)=\frac{f(x^*,t^*)L}{\rho AV^2} \tag{2}$$

Substituting the parameters of Eq. (2) into Eq. (1) gives the dimensionless form of the governing equation as

$$(w^*)''+N^*\frac{\partial^2 w^*}{\partial (x^*)^2}+K_0^*\frac{\partial^4 w^*}{\partial (x^*)^4}+K_1^*w^*+K_2^*(w^*)^3=f^*(x^*,t^*) \tag{3}$$

In which prime $(.)'$ denotes the derivatives with respect to dimensionless time t^*.The boundary conditions associated with Eq. (3) are in the form

$$w^*|_{x^*=0,1}=0\qquad \frac{\partial^2 w^*}{\partial (x^*)^2}|_{x^*=0,1}=0 \tag{4}$$

According to Galerkin's method, the solution of Eq. (3) can be expressed as

$$w^*(x^*,t^*)=\sum_{n=1}^{\infty}X_n(x^*)q_n(t^*) \tag{5}$$

where $X_n(x^*)$ is n-th mode shape and $q_n(t^*)$ is n-th modal coefficient. Because modal analysis of the beam and extracting mode shapes