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# Sensitivity analysis and Kriging based models for robust stability analysis of brake systems

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ABSTRACT

This paper presents a global strategy for the prediction of brake squeal. This approach is based on the global sensitivity analysis combined with Kriging modeling. The main aim is to assess the pertinence of using this strategy to build suitable and efficient instability predictors that can be potentially associated with numerical optimization schemes and/or robustness analysis algorithms for a robust design of brake systems. Through the use of a simplified brake system, the global sensitivity analysis is, firstly, shown to be essential for obtaining great insight on how design parameters influence individually and/or collectively the stability behavior. The latter is characterized by the distance of all the systems eigenvalues from the imaginary axis. It is shown that the so-called Sobol indices help for an objective quantification of the importance of taking parameter uncertainty into account in the whole design process. Based on these conclusions, Kriging modeling is, secondly, proposed to robustly predict friction induced instabilities. Its efficiency is then demonstrated. Consequently, the global sensitivity analysis and Kriging modeling give a very promising strategy helping for squeal prediction and, more generally, for optimal and robust design of brake systems.



## 1. Introduction

Brake squeal is of major importance in numerous industrial applications related to automobile, aeronautic and railway fields [1–4]. Although it does not affect enough the efficiency of braking, its high and audible frequencies and high intensities are often discomfort for passengers and may generate high warranty costs for car manufacturers [5]. So, it continues to be widely studied [5–7] with particular interests addressed to the good understanding of its phenomenology and appearance [2,8,9] and to the development of efficient strategies helping for its prediction [4,10–13]. Currently, a relative good knowledge of the brake squeal's phenomenology is available. Indeed, it is now common to define brake squeal as an acoustic event issued from nonlinear friction-induced vibrations occurring at friction interfaces between brake components [14]. Tribological and/or structural aspects have been considered to define models representing friction/contact laws explaining, at best, how these nonlinear vibrations appear. From the tribological point of view, there exist several models which suggest that friction induced vibrations are related to the dependence's type of the friction coefficient on the sliding speed between brake components [2,8]. While the stick-slip model defines a discontinuous law, the Stribeck model defines a smooth law giving the friction coefficient as an exponential function decreasing from the static to the dynamic friction coefficient and on a viscous term induced by the sliding velocity [15]. However, from the structural point of view, other models pointed out that friction-induced vibrations may occur even with a constant friction coefficient. This is the case for the sprag-slip and the mode coupling phenomena [2,8]. The latter, which this paper focuses, is defined by the situation where a couple of complex eigenvalues approach one another in frequency when the friction coefficient increases until a coalescence at the Hopf bifurcation point around which the corresponding real parts separate. When at least one real part becomes positive, self-friction induced vibrations and squeal noise can be generated [3,8,12]. The main problem is then to predict these self-excited vibrations. The most common techniques for predicting these non-linear behavior are based on numerical integration over time [3,12,16]. However, the use of these methods present drawbacks related to their considerable resources both in terms of computation time and data storage. Approximate methods and numerous techniques for model reduction can also be used [17–20]. However these still

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remain unsuitable especially to deal with robust and/or optimal design perspectives. In this framework, complex eigenvalues analysis (CEA) methods remain the most popular [10,11]. Based on the local linear stability analysis according to the Lyapunov theory, CEA based methods characterize squeal occurrences: if at least one real part of some eigenvalues is positive, the brake system becomes unstable and the associated imaginary part corresponds to the instability pulsation. Unfortunately, the CEA may lead to an under-estimation or an over-estimation of the unstable modes observed in the nonlinear time simulation due to the fact that linear conditions (i.e. the linearized stability around an initial equilibrium point with a defined contact state for each node) are not valid during transient oscillations [4,12,16]. Otherwise, CEA based methods remain less expensive than nonlinear transient dynamic analysis but are also still difficult to be coupled with robustness and optimization schemes. These difficulties are also related to the complexity of finite element models.

So, the main issue is to define suitable models capable to be faithfully representative of squeal phenomenon and efficient to apply robust analysis algorithms and/or numeric optimization schemes for designing brake systems. This paper is a part of this perspective. More accurately, it investigates preliminary capacities of the combination of the global sensitivity analysis from Sobol sense [21] together with Kriging formalism [22] to define efficient and useful alternative models to complex finite element models, which possess high potentialities to deal with robust stability analysis and thus with robust brake squeal prediction. Global sensitivity analysis is first discussed. In fact, despite the relatively high number of studies carried out for understanding how design parameters influence stability properties of brake systems [23–28] as the damping-induced destabilizing paradox [23,25,29–32] and for the quantification of uncertainties in the dynamic behavior of brake systems [33–39] with experimental tests [39,40], there are only few studies which propose to quantify with objective measures individual and collective impacts of design parameters on stability properties of brake systems. In this context, Butlin has analyzed, by using derivatives based computing, the sensitivity of the stability property of an idealized brake system to three main parameters namely, the contact-stiffness, the non-proportional damping and the velocity-dependent coefficient of friction [41]. He illustrated the fact that these three parameters affect the systems's stability in particular at boundaries between stability and instability zones and at the neighborhood of bifurcations occurring when the partial derivatives of the system's poles with respect to parameters are close to zeros. Otherwise, a parametric based sensitivity analysis of a brake system's stability can be found in [42]. Changes in friction coefficient and in lining stiffness are found to be impacting the stability boundaries. In this paper and by opposite to the mentioned studies, sensitivity analysis of a brake system's stability is globally dealt with (i.e. within the whole design space) by using a nonlinear method based on the so-called Sobol indices [21]. Indeed, based on computations of conditional expectations, Sobol indices give more objective measures on how the variability of one or several parameters combined together can influence the variability of the stability property modeled by the distance of the system's eigenvalues to stability/instability boundary defined by the imaginary axis in the complex plane. Sobol indices are then shown to be very helpful for ordering design parameters with respect to their impacts on the stability function defined. Indeed, high Sobol indices indicate that the corresponding parameters significantly impact the stability function while small indices denote weak influences. A key conclusion can be then stated about how it is important to take into account parameters uncertainties in the whole design process. Based on sensitivity analysis results, the second step consists of defining a suitable functional representation of stability properties. Indeed, as mentioned previously, CEA based on finite element models is still prohibitive. Thus, it is necessary to develop alternative simple models giving a compromise between their efficiency (cost of computing) and their aptitude to be representative of the stability behavior of the considered brake system. Kriging based models are suggested in this perspective. Indeed, as Kriging based models are the best linear non-biased estimators [22] and since they have shown interesting performances in numerous applications related to optimal design [43–45], they are proposed in this study to model robust stability properties depending on two sets of parameters: one set of random parameters driven by given probability density functions and another set including parameters that are assumed to be sufficiently controllable so that the deterministic character can be assigned. So, preliminary capacities of Kriging models to be efficiently and faithfully representative of stability properties of brake systems submitted to mode coupling instability are investigated and assessed in this paper.

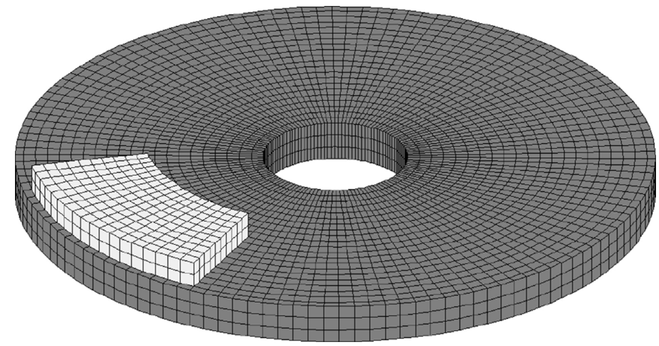

**Fig. 1.** Finite element model of the simplified brake system.

This paper is organized as follows. First, a simplified linear brake system is described in Section 2. Global sensitivity analysis from Sobol sense of brake system's stability is then performed and presented in Section 3. Kriging based modeling is described and considered to predict the brake system's instabilities in Section 4. Conclusion and perspectives are given at the end of the paper.

## 2. Finite element model

As previously said, we deal with stability analysis of brake systems from two points of view namely its sensitivity toward mechanical parameters and its efficient predictability. To achieve this objective, a simplified brake system is considered through its finite element model illustrated in Fig. 1. The latter consists of reduced finite element models of a pad and a disk one of which the contact surface is modeled by considering only nine uniformly spaced contact nodes for more simplicity. The inner surface of the disk is assumed clamped. Only a translational movement of the outline of the upper surface of the pad in the normal direction is possible whereas the rest of the upper surface of the pad is not constrained [36,46]. The simplified assembled model has 174 dofs issued from 80 dofs for the disk, 40 dofs for the pad and 54 dofs for the contact interface. Also, for the sake of simplicity and considering that the main objective of the presented study is to illustrate the global strategy based on sensitivity analysis and Kriging based modeling, material properties of the pad are supposed to be homogenous and no damping is included in the finite element model. In fact, it is well-known that damping can affect the stability of brake systems [25,26,29,31,47]: if the two modes involved in the coalescence are equally damped, damping stabilizes the system by shifting the eigenvalues toward the negative real parts. This effect is referred to as the “lowering effect”. As explained by Hoffmann and Gaul [29], if the two modes are not equally damped, the curves of the real parts become smooth with respect to the