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On the compression of a stack of truncated elastomeric cones as a nonlinearly responsive spring

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ABSTRACT

In an effort to construct a design tool for a mechanical spring featuring highly nonlinear spring stiffness, compression of truncated elastomeric cones has been studied using nonlinear finite element analyses involving neo-Hookean material law and contact elements. Series of finite element models of various geometric aspect ratios of truncated cones were calculated to form a fundamental database of the design tool. It was found that the compressive stiffness of the rubber cone can be non-dimensionalized with respect to the elastic modulus and a characteristic length of the cone. While the stiffness of the truncated rubber cone appears more linear between 0 and 5% of the compression ratio, the stiffness increases exponentially with progressing compression at higher compression ratios. Regression equations of the non-dimensional axial force and spring stiffness were obtained with reasonable accuracy, compared with the original finite element data.

1. Introduction

Mechanical springs with highly nonlinear force-displacement response are useful in many applications. In the area of truck suspension, an elastomeric pyramid was designed as a load cushion to provide a progressive spring rate [1]. A novel suspension strut was developed for providing constant bouncing frequency for a wide range of vehicle load [2]. In that work, to acquire the highly nonlinear spring stiffness much needed, the suspension strut was designed with a series of elastomeric cones stacked for compression. An innovative electromagnetic actuator was also developed to provide controlled movement that responds linearly to the input electric current [3]. To achieve that goal, the required highly nonlinear spring stiffness was again achieved by an elastomeric cone under compression. The nonlinear behavior of tensegrity prisms under compression has also received attention. Theoretical treatment has found that in the regime of large displacement, softening can happen as well as stiffening, depending on the aspect ratio of the structure, the applied prestress, and the material properties [4]. Experimental data have also verified the above theoretical findings [5]. All in all, structures with nonlinear stiffness can find many useful applications, and are worth further exploring.

It is the goal of this paper to document the effort toward constructing a design tool for creating a nonlinearly responsive spring based on rubber cones in series. Such stacks of rubber cones may work as a nonlinear cushion strut with sufficient stroke length and desirable spring characteristics. When a spring responds nonlinearly, although the spring stiffness varies as the deformation progresses, the combined spring stiffness of multiple springs in series can still be calculated based on the simple series law. As shown in Fig. 1, for any spring combination in series, the total displacement of the stack of springs is the summation of the deformation of each individual spring. Thus, the resulting force–displacement curve of the springs in series is the addition of the participating spring curves in the displacement at any given force. For example, while combining spring A and spring B in series, assuming the spring displacements d_i are nonlinear functions of the axial force F as,

$$d_a = f_a(F) \text{ and } d_b = f_b(F) \tag{1}$$

Upon combining the springs in series, the total displacement d_s is the sum of each individual displacement. And by differentiating the displacement by the force,

$$\frac{dd_s}{dF} = \frac{dd_a}{dF} + \frac{dd_b}{dF} \tag{2}$$

The series law for spring stiffness still holds as,

$$\frac{1}{k_s} = \frac{1}{k_a} + \frac{1}{k_b} \tag{3}$$

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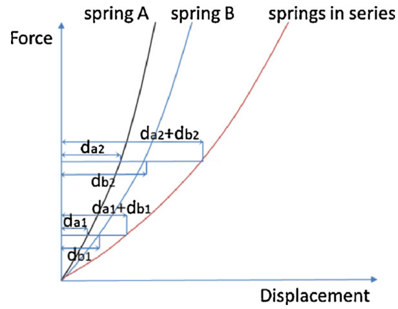

Fig. 1. Illustration of the series law applicable for nonlinear spring curves.

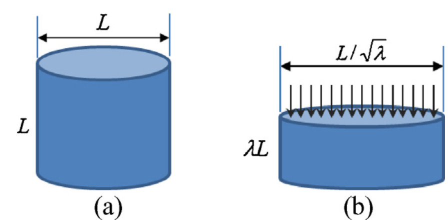

Fig. 2. Schematic of a cylinder under uniaxial compression: (a) original shape, (b) deformed shape with the shortened height being λL.

2. Similarity analysis

Similarity of the cone stiffness plays an important role in constructing a universal design tool. Closed-form solutions for a rubber cone under tension of a concentrated force at its apex were attempted [6]; however, for the compression case, no results have been published. To gain insight in this issue, simple compression of a rubber cylinder was investigated here. Assuming an incompressible neo-Hookean cylinder as shown in Fig. 2, under uniaxial compression of Δ, the height is compressed to be λL, and the diameter becomes $L/\sqrt{\lambda}$.

The strain energy function of the neo-Hookean material law of incompressible solid with $J = 0$ is,

$$W = C_1\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\right), \tag{4}$$

where C_1 denotes a constant equivalent to one-half of the initial shear modulus of the elastomer, and λ_i denotes the principal stretch ratios. Under uniaxial compression, the external work is equal to the internal strain energy as,

$$\int F d\Delta = W \cdot V = \frac{\pi}{4} C_1 \left[L(L^2 - 2\Delta L + \Delta^2) + \frac{2L^4}{L - \Delta} - 3L^3 \right] \tag{5}$$

The force can be derived by differentiating Eq. (5) as,

$$F = \frac{d(W \cdot V)}{d\Delta} = \frac{\pi}{4} C_1 \left[L(-2L + 2\Delta) + \frac{2L^4}{(L - \Delta)^2} \right] \tag{6}$$

Table 1
Dimensions of the three cases of cones for the similarity study. R is the large radius, r is the small radius, and h is the height of the truncated cone.

Case	R	r	h
1	5	0.5	5
2	10	1	10
3	20	2	20

A non-dimensional function is achieved as,

$$\frac{F}{L^2 C_1} = \frac{\pi}{2}\left(\frac{1}{\lambda^2} - \lambda\right) \tag{7}$$

The tangent stiffness is the slope of the force–displacement curve,

$$K = \frac{dF}{d\Delta} = \frac{\pi}{4} C_1 L \left[2 + \frac{4}{\lambda^3} \right] \tag{8}$$

A non-dimensional function for the stiffness can also be achieved as,

$$\frac{K}{LC_1} = \frac{\pi}{2}\left[1 + \frac{2}{(1-\delta)^3} \right], \tag{9}$$

where δ denotes the compression ratio, which is equal to $1 - \lambda$. Note that when Δ is very small, namely $\lambda \approx 1$, K/LC_1 in Eq. (9) becomes 1.5π, which approaches the small strain solution for compressed cylinders.

To verify the same similarity characteristics of Eqs. (7) and (9) on the compressed cones, finite element analyses were performed on three similar cones. The dimensions of the cones studied are shown in Table 1. Large strain finite element analyses involving incompressible neo-Hookean material and contact elements were adopted for the calculation. Detailed description of the finite element models is reported in Section 3. Results as shown in Table 2 reveal the same similarity characteristics previously found for compressed cylinders. By non-dimensionalizing the spring force with respect to the square of the large radius and C_1, the resulting values are almost identical at every compression ratio. Spring stiffness is also linearly proportional to the characteristic length of it.

3. Finite element analysis

To construct a database of the non-dimensional spring curves for a range of rubber cones, a series of finite element analyses were performed. 20 cases listed in Table 3 were calculated. The process of compressing each rubber cone up to 50% of its original height was simulated. The original height of every truncated cone was set at 1 cm. The small radius r varies from 0.1 cm to 0.5 cm. The large radius R varies from 0.8 cm to 3.2768 cm. The finite element models involved the hyper-elastic material model, large strain, and contact. Finite element software ANSYS 14.0 was used for the calculation with the nonlinear solution procedure turned on [7]. Neo-Hookean material law was used for its simplicity and ease to be directly linked to the Young's Modulus [8]. The compressibility parameter d

Table 2
Non-dimensional spring force and stiffness for the three cones studied for similarity characteristics.

Spring force				Spring stiffness			
Compress ratio (%)	F/R^2C_1			Compress ratio (%)	K/RC_1		
	Case 1	Case 2	Case 3		Case 1	Case 2	Case 3
2	0.044	0.044	0.043	1	2.20	2.19	2.17
10	0.260	0.260	0.260	4	2.70	2.70	2.70
20	0.704	0.704	0.703	15	4.44	4.44	4.44
30	1.497	1.497	1.497	25	7.94	7.94	7.94
40	3.080	3.080	3.080	35	15.83	15.83	15.83
50	6.971	6.972	6.972	45	38.91	38.91	38.91