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Pattern formation in the three-dimensional deformations of fibered sheets

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A B S T R A C T

We simulate pattern formation in the deformations of a pantographic lattice using a model of elastic surfaces that accounts for the geodesic bending of the constituent fibers. The theory predicts an unusual arrangement of coexistent phases observed in an actual lattice, manufactured by a 3D printing process, in which the fibers undergo part-wise uniform shears separated by internal transition layers controlled by geodesic bending stiffness.

1. Introduction

In this work we apply a recently developed two-dimensional continuum theory of elastic surfaces [38] to model the main features of the deformations observed in pantographic lattices. The lattice is composed of intersecting fibers, or rods, that form two orthogonal families in an unstressed reference plane. Each member of a given fiber family – regarded as a collection of parallel material curves – intersects every member of the second family at internal pivots. The resistance of these pivots to rotation about an axis aligned with the surface normal is modeled as elastic resistance to a change of angle, or shear, of the intersecting fiber families. The model also accommodates fiber stretch and three-dimensional deformations of the lattice surface. In deformations that involve a change of surface shape, the convected fibers of the pantograph flex to conform to the evolving embedding geometry. This is modeled by assigning strain energy to the normal curvatures and twists of the fibers. These variables, in turn, are determined by the second fundamental form (the curvature) of the surface, and so in this respect the present model is similar to conventional plate theory. However, in contrast to the conventional framework, the present model also accounts for the geodesic bending of the fibers. This mode of deformation occurs when, for example, the lattice is bent while deforming in a fixed plane. In the continuum theory, the effect is associated with geodesic curvature of the fibers, which in turn is controlled by the metric geometry of the surface. In particular, it involves the metric and its spatial derivatives via the Levi-Civita connection coefficients. Geodesic bending is therefore a strain-gradient effect. The pantographic substructure thus furnishes an explicit realization of strain-gradient elasticity [39,23,22,17,8,9,14].

In Section 2 we discuss deformations observed in the so-called bias test of an actual pantographic sheet manufactured by 3D printing. These reveal a remarkable pattern of distinct phases in which the fiber shear is nearly uniform, separated by narrow internal transition layers exhibiting pronounced geodesic bending. Section 3 is devoted to a synopsis of the continuum theory developed more fully in [37,38]. This framework may be viewed as a further development of a line of research on the continuum mechanics of fibrous materials initiated by [32,19] and [25,26], concerning surfaces with perfectly flexible embedded fibers [42,43]; for inextensible fibers with flexural resistance; and more recent developments in the setting of generalized bulk continua [35–37,5,24], biological materials [13,20] and anisotropic mixtures [29,30]. In Section 4 we discuss the details of a finite-element solution procedure and illustrate the model with a simulation of the phase coexistence observed in the bias test, regarded as an in-plane deformation. Finally, in Section 5 we exhibit a simulation of a three-dimensional deformation that

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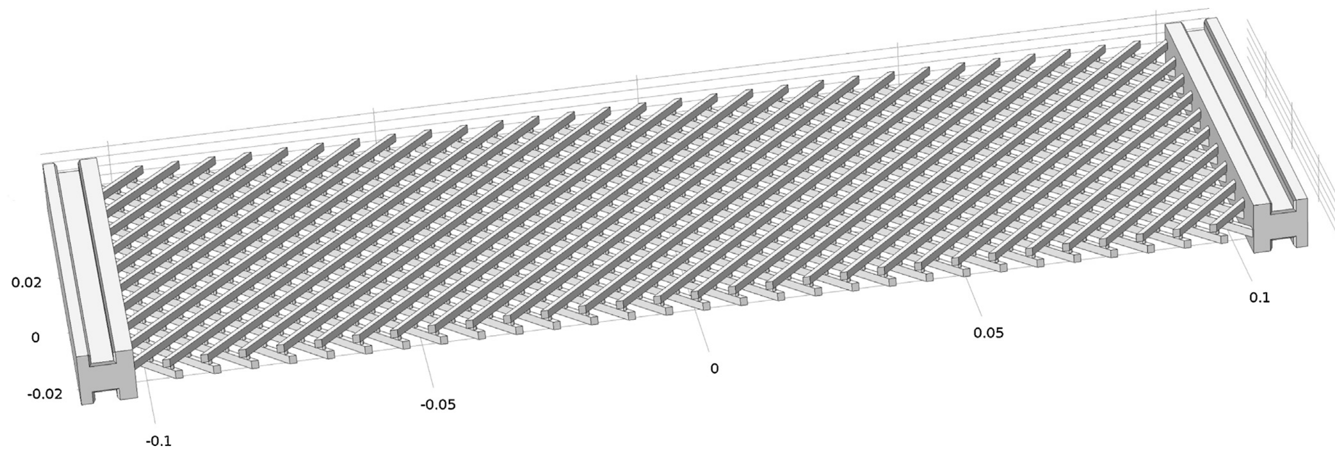

Fig. 1. Pantographic lattice topology.

induces a similar internal strain pattern combined with substantial fiber twist.

A difficult unresolved problem, not addressed here, concerns the modeling of the complex internal structure of the lattice and the implementation of a suitable homogenization procedure leading to an appropriate continuum theory [1,34]. However, to the extent that the present theory faithfully predicts the details of the observed phenomenology, it may be viewed as furnishing an appropriate target model for deeper investigations of this kind.

2. Observations on pattern formation and phase segregation in the bias test of a uniform pantographic lattice

Fig. 1 depicts a pantographic lattice topology generated by SolidWorks. The same software controls the 3D printer FORMIGA P100 (EOS GmbH), which was used to construct specimens for extensional bias testing. The material used was PA 2200 (polyamide powder). The geometry of the specimen was specified by means of the STL format files (i.e., a standard file format broadly used for rapid prototyping, 3D printing and computer-aided manufacturing), which provided the input to the 3D printer. In Fig. 2 the periodic cell of the printed specimen is shown in detail.

We remark that the process of 3D printing allows for the contextual construction of the specimen to be tested and the clamping devices used for connecting it to the testing apparatus.

The terminal pivots in the structure connect the terminal ends of the fibers meeting at the external boundary of the lattice. The pivots are realized by means of short connecting cylinders whose dimensions can be varied to endow the lattice with a suitably tuned shear elastic energy.

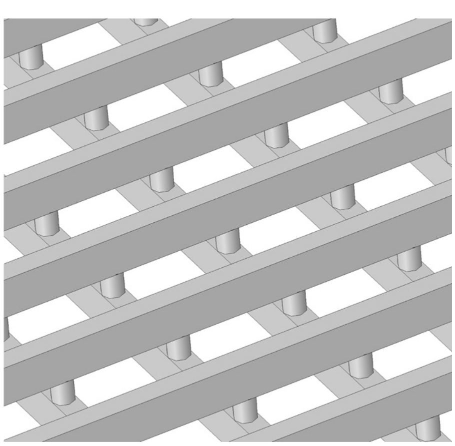

Fig. 2. Details of the periodic cell of the printed specimen.

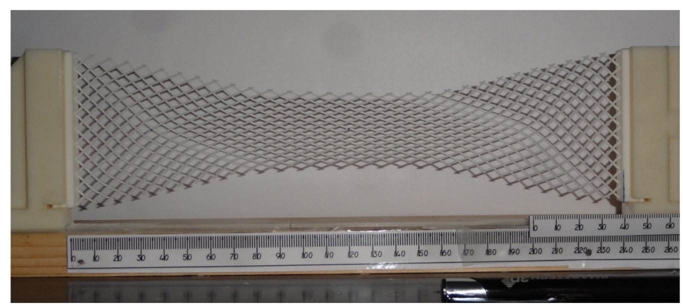

Fig. 3. An example of bias extension test for pantographic lattice with coarse mesh.

On the basis of a simplified 2D continuum model for a lattice with inextensible fibers [11], it is anticipated that in the extensional bias test two triangular regions bounded by fibers remain rigid and that distinct phases exhibiting piecewise constant shear deformations emerge. This is precisely what is observed (see Figs. 3 and 4.

It is further observed that the shear deformation is segregated into distinct coexistent phases separated by internal transition layers in which geodesic bending predominates. Also, some of the fibers exhibit substantial stretching, the modeling of which calls for an extended theory of the kind adopted here.

In Fig. 3 the physical mesh is relatively coarse and the ratio between the rigidity of the pivots and the bending stiffness of the fibers is such that the thickness of the transition layers is limited to approximately three times the mesh size.

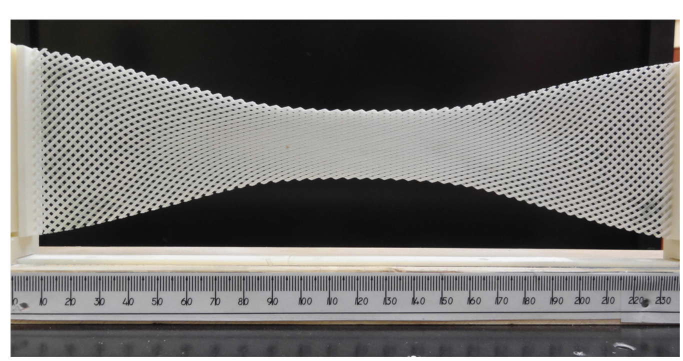

Fig. 4. An example of bias extension test for pantographic lattice with finer mesh.