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Effect of particle size distribution on fracture toughness of polymer composites considering plastic void growth after particle debonding

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ABSTRACT

Fracture toughness of particle reinforced polymers is affected by the size distribution of particles. Dissipation mechanisms, such as particle debonding, matrix shear bands or plastic voiding, may be responsible for this behaviour. It was examined whether matrix voiding energy after particle debonding from the matrix depends on particle size distribution. The stress field solution of the mechanical problem of a spherical particle within a spherical elastic/perfectly plastic matrix under hydrostatic tensile stress was used. After particle debonding, the yielding energy of the matrix shell around a single micro- or nano-particle was calculated. Applying a general model for the calculation of toughness together with an assumed particle size distribution function allowed the examination of the influence of parameters of the size distribution functions. The fracture toughness increases with increasing mean particle diameter and is highest for composites with wide particle size distribution functions, i.e. with larger standard deviations.

1. Introduction

Throughout the last decades the subject of improving the mechanical properties of particle filled polymers received great attention and a large body of publications exists. A survey about this development was given by Fu et al. [1]. The particle size plays a decisive role among parameters that influence toughness (see, for example, Singh et al. [19]). Depending on the particle or matrix materials used, toughness may increase or decrease with changing particle size distribution parameters. Modelling of toughness for composites with particles of mean size was proposed for example by Evans et al. [2], Huang and Kinloch [17], Hsieh et al. [3], Williams [4], Lauke [5] and Lauke and Fu [6]. The effect of particle size distribution for the debonding mechanism was first considered by Evans and Faber [7] and was described for different debonding criterions by Lauke [8]. Debonding is a necessary precondition for subsequent matrix plastic voiding. The debonding stress at the particle/matrix interface increases with decreasing particle size. This dissipation mechanism was considered in that paper for Gaussian- and Lognormal particle size distributions. A single particle in the stress field in front of the crack which increase towards the crack tip was considered. Consequently the debonding energy density of particles at a certain position is a function of the particle size distribution. The total debonding energy was calculated by integration of the debonding energy density over the distance coordinate. The plastic void growth mechanism was modelled by Williams [4], Zappalorto et al. [9] and Lauke [10]. The model developed in the last reference is extended herein to the case of nano- or micro-particles with a size distribution. After particle debonding this mechanism occurs and the corresponding energy density depends on the particle size, which must be considered in the calculation. For the calculation of the total dissipation energy the integration over the stress in front of the crack was applied to circumvent problems with the integration over the distance coordinate; applying this procedure for the calculation of the plastic void growth mechanism would lead to a transcendental equation for the crack resistance.

The integration of energy density over the stress under consideration of particle size distribution, which eventually provides an analytical equation for the composite fracture toughness, is the new point of the modelling herein. Sections 2 and 3 will concisely summarize main equations to make it throughout understandable.

2. Composite fracture toughness

The energy necessary to initiate crack propagation is called crack resistance, R_c, or fracture toughness. During crack growth, the crack consumes energy, R_{pz}, to form the new fracture surface

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Nomenclature

Notations

A_{ij}^k	stress concentration factors
d_p	particle diameter
$d_{p,\mathrm{mean}}$	mean particle diameter
$d_{p,\mathrm{min}}$	minimum particle diameter
$d_{p,\mathrm{max}}$	maximum particle diameter
E_c	composite modulus
E_p	particle modulus
E_m	matrix modulus
F_n	function $F_n = f_n d_p^3$
f_N	probability density
f_n	normalized probability density
G_d	specific debonding energy at the particle/matrix interface
N_p	number of particles
n_p	particle volume density
r, θ, ϕ	spherical coordinats, origin at the particle centre
r_p	particle radius
r_y	radius of the yielding region in matrix
r_0	radius of the composite element
R_c, R_m	crack resistance of composite and matrix (energy per unit area of crack)
R_{dz}	specific dissipation zone energy (energy per unit area of crack)
R_{pz}	specific process zone energy (energy per unit area of crack)
s	ratio of hydrostatic stress to matrix yield stress, $s = \sigma_0/\sigma_{\mathrm{my}}$
s_1, s_2	integration limits for the normalized hydrostatic stress, s
s_d	normalized hydrostatic stress at debonding, $s_{\mathrm{d}} = \sigma_{0,\mathrm{d}}/\sigma_{\mathrm{my}}$
$s_{\mathrm{min}}, s_{\mathrm{max}}$	minimum and maximum normalized hydrostatic stress, s
s_N	standard deviation of the Gaussian normal distribution
s_{LN}	standard deviation of the Lognormal distribution
Δu	difference between the total and elastic radial displacement at $r = r_0$
v	particle volume fraction
$\tilde{v}$	particle fraction within the composite elelment, $\tilde{v} = (r_p/r_0)^3$
v_m	volume fraction of matrix
V	composite volume
V_p	total particle volume
W_{my}	matrix yielding energy of the matrix shell of one particle
β	shape factor of dissipation zone
η_{my}	yielding energy density (energy per volume)
κ	stress concentration factor, $A_{rr}^p = A_{rr}^m = \kappa$
μ_N	expected value of particle size of Gaussian normal distribution
μ_{LN}	expected value of particle size of Lognormal distribution
v_m, v_p	Poisson's ratio of matrix and particle, respectively
ρ, φ, z	radial, angular and depth coordinates with the origin at the crack tip
ρ_y	width of dissipation zone for matrix yielding
σ_0	hydrostatic tensile stress at the outer surface, $r = r_0$, of the composite element
$\sigma_{0,\mathrm{min}}$	minimum hydrostatic stress where plastic yielding in the matrix shell around a particle starts
σ_d	debonding stress at particle/matrix interface, corresponds to the maximum radial stress component
σ_{my}	matrix yield stress in uniaxial tension
$\sigma_r^{my}, \sigma_\theta^{my}$	radial and hoop stresses in yielding matrix
$\sigma_{rr}^p, \sigma_r^p$	radial stress in particle
$\sigma_{rr}^m, \sigma_r^m$	radial stress in elastic matrix

in the process zone. At the same time energy, R_{dz}, is dissipated by matrix yielding around debonded particles within a larger zone of width, $2\rho_y$, subsequently called the yielding zone, see Fig. 1. The crack spreads over a small area dA of unit thickness. Summation of the process zone energy, given by the product of matrix toughness, R_{m}, and the relevant volume fraction, v_{m}, and the dissipation zone energy, as the integral over all local contributions, provides composite fracture toughness:

$$R_c = R_{pz} + R_{dz} = R_m v_m + 2\int_0^{\rho_y} \eta_{my}(\rho)\, d\rho \tag{2.1}$$

where η_{my} is the volume specific matrix yielding energy, ρ is the distance coordinate from the crack tip.

Under remote mode I loading of a specimen a multiaxial stress field is developed in front of a crack, which can be written as a function of the coordinates (ρ, φ) as: $\sigma_{c,ij} = (R_c E_c/\rho)^{1/2} g_{ij}(\varphi)$, $(i, j = x, y, z)$ with E_{c} as the Young's modulus of the composite.

To simplify the following derivations it was assumed that this multiaxial stress state at the position ρ can be approximated by a hydrostatic tensile stress, σ_0:

$$\sigma_0(\rho) = \left(\frac{\beta R_c E_c}{\rho}\right)^{1/2} \tag{2.2}$$

where β, as a zone shape and size factor, can be used as a fitting parameter. A similar approach was used by Zappalorto et al. [9], who argued that the hydrostatic stress component of the crack tip stress field is of major importance for such analysis.

The parameter β was used to consider this quantitative deficiency in the true values of stresses.

Consequently the half width of the dissipation zone, ρ_y, is given by:

$$\rho_y = \frac{\beta R_c E_c}{\sigma_{0,\mathrm{min}}^2} \tag{2.3}$$

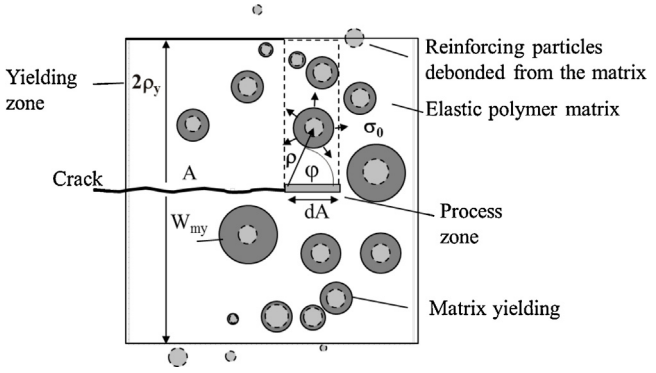

Fig. 1. Cross sectional view of dissipation zone in front of the crack; with A: crack area, dA: crack extension, (ρ, φ, z): cylindrical coordinates of particle location (z perpendicular to the plane), W_{my}: matrix yielding energy around one particle.