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Pull-in voltage analysis of electrostatically actuated MEMS with piezoelectric layers: A size-dependent model

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ABSTRACT

A size-dependent model for electrostatically actuated microbeam-based MEMS (micro-electro-mechanical systems) with piezoelectric layers attached is developed based on a modified couple stress theory. By using Hamilton's principle, the nonlinear differential governing equation and boundary conditions of the MEM structure are derived. In the newly developed model, the residual stresses, fringing-field and axial stress effects are considered for the fixed–fixed microbeam with piezoelectric layers. The results of the present model are compared with those from the classical model. The results show the size effect becomes prominent if the beam dimension is comparable to the material length scale parameter (MLSP). The effects of MLSP, the residual stresses and axial stress on the pull-in voltage are also studied. The study may be helpful to characterize the mechanical and electrostatic properties of small size MEMS, or guide the design of microbeam-based devices for a wide range of potential applications.

1. Introduction

Micro-scale mechanical structures, whose characteristic sizes are in the order of micron or sub-microns, are extensively utilized in micro-electromechanical systems (MEMS), e.g., micro-switches and micro-resonators.

Typical electrostatically actuated MEMS are comprised of a conductive deformable electrode suspended above a fixed electrode (Fig. 1). An applied direct current (DC) voltage between the two electrodes results in the deflection of the deformable electrode toward to the fixed electrode due to electrostatic attraction. As the voltage increases beyond a critical value, the movable electrode becomes unstable and collapses onto the fixed electrode. This phenomenon, known as pull-in instability, is important in designing MEMS structures. The critical displacement and the critical voltage associated with this instability are referred to as the pull-in displacement and the pull-in voltage, respectively. In micro-resonators the designer should avoid such instability in order to achieve stable motions; however, in micro-switching applications the designer takes advantage of this effect to optimize the performance of the device. To design the electrostatically actuated MEMS structures, many papers have been published to model the electro-mechanical coupled problem and to control and optimize the pull-in voltage [1,2].

In the electrostatically actuated MEMS, there are many factors that may influence the pull-in voltage, such as the residual stress in thin film, axial stress, fringing field and even the effect of Casimir force [3] and Van der Waals force [4]. The residual stress originates from the mismatch of both thermal expansion coefficient and crystal lattice period between substrate and thin film [5]. The residual stress is very important to precisely design MEMS and it can be tensile or compression in surface micromachining techniques. In actual situation, on the other hand, due to the finite width of the plate, a uniform electric field cannot drop abruptly to zero at the edge and there is always a "fringing-field" existing at the edges of two electrodes. Duan et al. [6] investigated the influence of the fringing field in the cantilever NEMS actuator, and pointed out that the fringing field effect increases the pull-in deflection and decreases the pull-in voltage. Furthermore, surface effect is also found to affect the pull-in instability of micro/nano-structures [7,8].

However, the main drawbacks of electrostatically actuated microsystems are high driving voltage and low reliability [9,10]. Piezoelectric materials are getting increased attention since piezoelectric materials can generate very precise small motions and have a significant characteristic of high force transmission. Besides, piezoelectric actuators also enjoy advantage such as light

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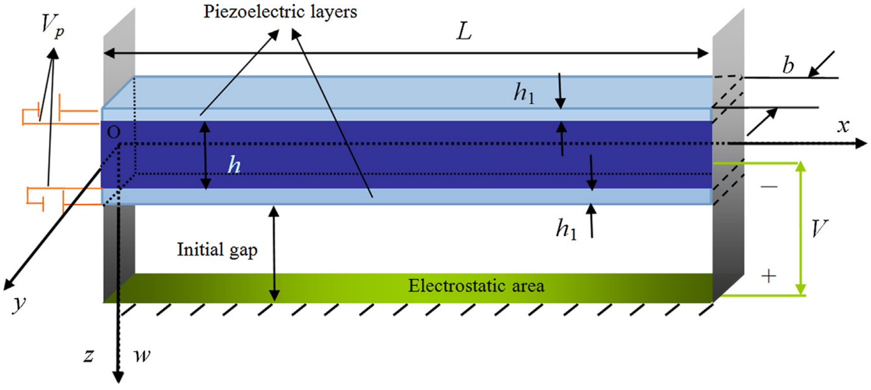

Fig. 1. Schematic of fixed–fixed MEMS actuator with piezoelectric layers.

weight, rapid response, high operating bandwidth and low power consumption, which is suitable in MEMS applications. Therefore, a new idea on the design and improve the precision of pull-in voltage is implemented to drive the micro-structure with hybrid piezo-electrostatic excitation in micro-systems in order to achieve greater efficiency or to control pull-in voltage [11]. Rezazadeh et al. used piezoelectric materials to enhance the controlling of MEMS structure behaviors such as pull-in voltage by attaching piezoelectric layers on the upper and lower beam [12].

On the other hand, miniaturized beam are the core structures widely used in MEMS [13] and has attracted much attention from the scientific community. For applications, the characteristic dimension (typical the thicknesses) of the beam are typically on the order of microns or even sub-microns. In the past decades, many theoretical studies of microscale beams were carried out based on the classical continuum theory [2,14]. Though the wide applications for the classical continuum theory, the length scales associated with material's microstructure are often sufficiently small to call the applicability of classical continuum models into question because more and more experimental results reveal that size-dependent mechanical and electric characteristics for the micro-structures indeed exist in different materials [15–19]. Lacking internal MLSP, classical beam models cannot be used to interpret these microstructure-dependent size effects. Recently, a modified couple stress theory was proposed [20], in which the couple stress tensor is symmetric and only one internal MLSP is involved, to predict the size effect of the linear and nonlinear Bernoulli–Euler beam [21–23], the linear and nonlinear Timoshenko beam [24,25], Linear functionally graded Euler–Bernoulli beam [26], Timoshenko beam [27], the Kirchhoff plate [28] and pull-in phenomena in MEMS [29–31]. For the characteristic size of the electrostatically actuated MEMS with piezoelectric layers, its characteristic length is in the order of micron or sub-micron, and it is reasonable to believe that size effect exists in the micro-structures and non-classical theory should be employed to study such size effect, which has not yet been studied to the best of our knowledge.

The paper aims to close the aforementioned gap by establishing a versatile size-dependent model for pizeo-electro-mechanical coupling problem, with application to the size-dependent pull-in instability under mechanical and electrostatic forces. The outline of this paper is organized as follows. In Section 2, the variational formulations of the size-dependent model based on the modified couple stress theory are in detail deduced by using the Hamilton's principle. Subsequently, in Section 3, Galerkin method is introduced to solve the problem and the size-dependent pull-in voltage is then studied and discussed. Finally, some major conclusions are summarized in Section 4.

2. Formulations

Based on the modified couple stress theory [20], the bending strain energy density is a function of both strain and curvature. Then the bending strain energy U_m in a deformed isotropic linear elastic material occupying region Ω is given by

$$U_m = \frac{1}{2}\int_{\Omega} (\boldsymbol{\sigma} : \boldsymbol{\varepsilon} + \boldsymbol{m} : \boldsymbol{\chi})\,\mathrm{d}v \tag{1}$$

where $\boldsymbol{\sigma}$ is the stress tensor, $\boldsymbol{\varepsilon}$ is strain tensor, $\boldsymbol{m}$ is deviatoric part of the couple stress tensor and $\boldsymbol{\chi}$ is symmertric curvature tensor defined as follows:

$$\boldsymbol{\sigma} = \lambda \mathrm{tr}(\boldsymbol{\varepsilon})\mathrm{I} + 2\mu\boldsymbol{\varepsilon} \tag{2}$$

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left[\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T\right] \tag{3}$$

$$\boldsymbol{m} = 2l^2\mu\boldsymbol{\chi} \tag{4}$$

$$\boldsymbol{\chi} = \frac{1}{2}\left[\nabla\boldsymbol{\theta} + (\nabla\boldsymbol{\theta})^T\right] \tag{5}$$

where the λ and μ are Lamé's constants, l is internal MLSP, $\boldsymbol{u}$ is the displacement vector and $\boldsymbol{\theta}$ the rotation vector given by

$$\boldsymbol{\theta} = \frac{1}{2}\mathrm{curl}\boldsymbol{u} \tag{6}$$

On the other hand, the variation of the total work done by external forces is

$$W = \int_{\Omega} \left(\boldsymbol{f}\cdot\boldsymbol{u} + \boldsymbol{c}\cdot\boldsymbol{\theta}\right)\mathrm{d}v + \int_{\partial\Omega} \left(\boldsymbol{t}\cdot\boldsymbol{u} + \boldsymbol{s}\cdot\boldsymbol{\theta}\right)\mathrm{d}v \tag{7}$$

where $\partial\Omega$ is the surface of Ω and $\boldsymbol{f}$ is the body force, $\boldsymbol{c}$ is the body couple, $\boldsymbol{t}$ is the traction, $\boldsymbol{s}$ is the surface couple, respectively.

Consider a straight beam with piezoelectric layers attached at the top and bottom surfaces, as shown in Fig. 1. The beam, with thickness h, width b, length L, density ρ, and Young's modulus E, has a pair of piezoelectric layers bonded on its surfaces. Each of the piezoelectric layers has thickness $h_1 (h_1 \ll h)$, density ρ_0 and Young's modulus E_p. The piezoelectric layers are located throughout the beam width and length, as well as complete bonding is assumed, where x-axis is coincident with the centroidal axis of the undeformed beam, y-axis is the neutral axis and z-axis is the symmetry axis. Thus, the loading plane coincides with the xz plane, and the cross-section of the beam parallel to the yz plane. A voltage V is applied between the movable beam and the fixed electrode, and V_p is applied between the top and bottom surfaces for each piezoelectric layer.