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Comparative study on peak stress multipliers for perforated flat plate with various loadings

Jianfeng Mao, Shiyi Bao*, Lijia Luo, Zengliang Gao

Institute of Process Equipment & Control Engineering, Zhejiang University of Technology, Hangzhou 310014, China

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ABSTRACT

In this study, a mathematical approach is proposed to calculate the stress distributed around the circular hole of perforated flat plate. This analytical method allows an accurate evaluation of the stress state in the perforated plate. Besides, finite element analysis is carried out to determine the stress for perforated plate with different ligament efficiencies, various biaxiality ratios. After that, one dimensionless parameter named as peak stress multiplier (PSM) is analyzed with varying the ligament efficiency and biaxiality ratio for both numerical and analytical method. Through rigorous analysis, some important conclusions are summarized on the PSM. It is found that the PSM is not located at centric hole, and the one predicted by FEM is conservative.

1. Introduction

Perforated flat plate (also known as 'tube sheet') is very important structural element in various processing apparatus and equipments. A typical perforated flat plate is shown in the following Fig. 1, which is installed in a nuclear facility. However, when designing this kind of structure, many engineers encounter huge difficulty due to its complex geometry and loading patterns [1–3]. For instance, some perforated flat plate contains hundreds of tubes or holes, so how to deal with this problem attracts many researchers. As well known, the circular hole of perforated flat plate may be arranged in various patterns. For simplicity, there are usually two typical arrangements for the tubes (see following Fig. 3), one of which is rectangular arrangement, and the other is arranged in a triangular pitch. In correspondence with the tube arrangement, the loading pattern is another important factor to determine the maximum stress [4–6]. In order to reflect the geometric and loading influences on the perforated flat plate, two parameters labeled as ligament efficiency and biaxiality ratio are raised and taken into account in designing process.

Since the perforated flat plate is so crucial for safety operation of the related apparatus, numerous efforts have been paid to successfully design perforated flat plate. For this reason, ASME code, Section 3 presents a method of analysis for perforated flat plate by utilizing the concept of the equivalent solid plate, but does not provide any analytical solution and deduction process of the effective stress equations [7]. In order to successfully apply the ASME code, the effective material properties of perforated shell are still calculated by FEM with respect to the ligament efficiencies [8]. The elastic stress fields of the thickness perforated plate subjected to the biaxial load are systematically studied by FEM [1–3,9,10]. Although some verification have been given for thin perforated plates by FEM, the peak stress multipliers (PSM) cannot be simply used for the thick plates when the ratio of the thickness (t) to pitch (p) is equal to or greater than 2 [11]. For general perforated structure, extensive studies are performed on investigating the influence of geometric parameters and loading patterns on PSM [12,13], and the numerical results are compared and validated with the experimental data [14]. For some simplicity and efficiency reasons, the analytical investigation is used to study the stress state of plates with different central cutout [15,16]. Accordingly, exact solution for stresses of perforated plate is investigated by using Airy stress function, and the stress concentration factors are analyzed for several types of in plane loading [17,18]. In studying the stress distributions of the multiple circular holes with the rhombic pattern, the alternating method is proposed, the analytical solution of which correlates with a successive iterative superposition process [19]. As well known, most perforated flat plates do not have analytical solutions for stress analysis, so FEM is very desirable. Through FE simulation, the empirical equations for the stress concentration around circular hole are proposed, and the agreement between FEM and empirical equations is achieved [9,20]. Due to the cutout effect, the cutout-strengthening of perforated plates is discussed for different type of loading, and the effects of many parameters on stress

* Corresponding author. Tel.: +86 0571 88320907; fax: +86 0571 88320842.
E-mail address: bsy@zjut.edu.cn (S. Bao).

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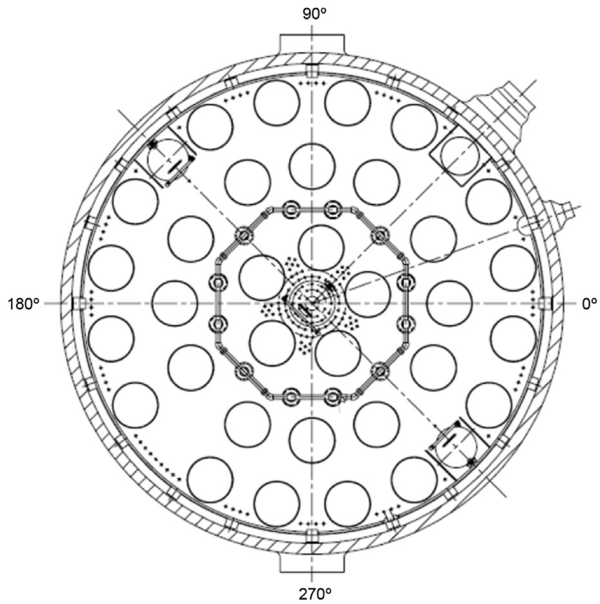

Fig. 1. Schematic of a perforated flat plate applied in a steam generator.

are analyzed covering the cutout shape, size, location, angle and so on [1–3,21,22]. To overcome some limitations discussed above, the ultimate strength characteristics of perforated flat plate are investigated under combined biaxial loads [1–3,23]. Through comparison between von Mises criterion and the flow rule chosen, the size of yield surface as well as the shape is found to depend on the ligament efficiency [24]. In dealing with some special issue, subroutine programs are developed to implement the associated algorithms in the commercially available FE package [25]. Accordingly, several approaches have been considered to illustrate the influences of the parameters on stress multiplier (SM), among them are mainly numerical approaches and analytical approaches. A SM is defined as the ratio of the maximum principal stress to the nominal stress under a given loading pattern, and is widely used in verifying the design of perforated flat plates.

In designing a perforated flat plate as shown in Fig. 1, the simplification is very important for both the geometry and loads. Essentially, the design works can be reduced to the studies on the peak stress multiplier (PSM), which is presented in the paper. In order to contain the two directions for loading on respective hole arrangement, the perforated plate needs at least seven circular holes, including the centric hole. Accordingly, the analysis is carried out on such perforated plate. The geometric parameter is described as ligament efficiency, the effects of which on PSM are exploited in detail. The loading pattern is generalized as biaxiality ratio, the effects of which on PSM are also investigated in depth. In achieving the investigation, the analytical approach is established to compare with the FEM. Comparative studies are performed on PSMs for perforated flat plate with various loadings. Finally, it is found that the FE results are in roughly good agreement with the analytical solutions, and the FE predication on the PSM is conservative.

2. Mathematical models of perforated flat plate

2.1. Analytical model

As described in Fig. 2, the equilibrium state of plate element contained halves of the holes is established in the paper. The analytical model for perforated flat plate is developed base on the equilibrium state of internal forces acting on the isolated plate element. Compared with traditional theory of plate shell, it is noted that the circular hole results in material discontinuity that leads to local disequilibrium on every single hole. However, the counter force/moment from other hole brings the wedge-shaped element of the perforated flat plate to be equilibrium. Accordingly, this wedge-shaped element is regarded as inseparable smallest element for perforated flat plate.

According to this concept on wedge-shaped element, the differential equation can be deduced as that of unperforated solid plate as follows,

$$\frac{dm_r(r)}{dr} + \frac{m_r(r) - m_\theta(r)}{r} = -t(r) \tag{1}$$

where $m_r(r), m_\theta(r)$ are the moment intensity along respective direction. $m_r(r)$, $m_\theta(r)$ have their own expressions,

$$m_r(r) = D_r\left(\frac{d\varphi}{dr} + \nu\frac{\varphi}{r}\right), \quad m_\theta(r) = D_\theta\left(\nu\frac{d\varphi}{dr} + \frac{\varphi}{r}\right) \tag{2}$$

where D_r is modified plate stiffness in the radial direction, D_θ is the one in the circumferential directions, φ is the angle of plate deflection. D_r, D_θ are written as,

$$D_r = \frac{Et^3}{12(1-\nu^2)}\cdot\frac{r(\Delta\theta) - d}{r(\Delta\theta)}, \quad D_\theta = \frac{Et^3}{12(1-\nu^2)}\cdot\frac{\Delta r - d}{\Delta r} \tag{3}$$

where t is the thickness of the plate, ν is the Poisson ratio, $\triangle r$, $\triangle\theta$ are finite element increments along radial and circumferential direction respectively. $r\triangle\theta$ is the pitch of holes, d is the hole diameter, E is the Young modulus. Based on the elemental geometry of the perforated flat plate in Fig. 2, two weakening coefficients are proposed in radial and circumferential direction respectively, they are written as $C_r = [r\Delta\theta - d]/r\Delta\theta$, $C_\theta = [\Delta r - d]/\Delta r$. By substituting Eqs. (2) and (3) into Eq. (1), the equation of equilibrium of internal force can be induced as,

$$\frac{d^2\varphi}{dr^2} + \left(1 + \nu - \nu\frac{C_\theta}{C_r}\right)\frac{d\varphi}{rdr} - \frac{C_\theta\varphi}{C_r r^2} = -\frac{1}{D_r}t(r) \tag{4}$$

where $t(r)$ is the function of transverse force intensity, C_θ and C_r are weakening coefficient on section π_θ and π_r respectively. Different loading pattern can be reflected on $t(r)$, accordingly, it can be written as,

$$t(r) = k_1 r + \frac{k_2}{r} + k_3 r^2 \tag{5}$$

where k_1, k_2, k_3 are the coefficient with respect to loading patterns. The solution of Eq. (1) can be yielded as,

$$\varphi(r) = -\frac{1}{D_r}\left[\frac{k_1 r^3}{(3-\lambda_1)(3-\lambda_2)} + \frac{k_2 r}{(1-\lambda_1)(1-\lambda_2)} + \frac{k_3 r^4}{(4-\lambda_1)(4-\lambda_2)}\right] + C_1 r^{\lambda_1} + C_2 r^{\lambda_2} \tag{6}$$

where C_1 and C_2 are integration constants, while λ_1 and λ_2 can be obtained as the roots of the following characteristics equation:

$$\lambda^2 + \nu\left(1 - \frac{C_\theta}{C^r}\right)\lambda - \frac{C_\theta}{C^r} = 0 \tag{7}$$

Under a given $\varphi(r)$, $m_r(r)$ and $m_\theta(r)$ are proceed to be determined accordingly. After that, Eq. (1) allows us to compute stresses on the perforated plate element (see Fig. 2). $\sigma_r(r)$, $\sigma_\theta(r)$ located at a distance along z direction from the neutral plane are calculated with the following equations,

$$\sigma_r(r) = \frac{12}{t^3}m_r(r)z, \quad \sigma_\theta(r) = \frac{12}{t^3}m_\theta(r)z \tag{8}$$