Mechanics Research Communications 66 (2015) 49–59

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Response of moving load due to irregularity in slightly compressible, finitely deformed elastic media

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ARTICLE INFO

Article history:
Received 24 September 2014
Received in revised form 12 February 2015
Accepted 27 March 2015
Available online 4 April 2015

Keywords:
Slightly compressible
Moving load
Irregularity
Anisotropy
Normal stress
Shear stress

ABSTRACT

The present paper has been framed to study the stresses produced on the rough surface of a slightly compressible, finitely deformed half space due to a normal moving load. The surface of the medium is irregular with parabolic type of irregularity. The perturbation method is applied to find the displacement field. The normal and shear stresses have been obtained in closed form and discussed numerically by means of figures. It has been observed that the shear stress developed at different depths below the surface depends on the irregularity depth, frictional coefficient and irregularity factor of the rough surface of the medium. Also, surface plots have been drawn to analyze the combined variation of non-dimensional stresses and irregularity factor against depth.

1. Introduction

The incompressible and slightly compressible anisotropic materials have drawn attention recently due to interest in the effects of large primary static deformations on dynamical material response. Although stress is often induced in the formation of layers of material by industrial manufacturing processes such as fabrication, but a far greater concern within many structures is the stress arising from external loads. Material technology now enables manufacture of materials able to withstand large deformation and support high external load prior to failure. In particular, it is noted that the use of rubber-like components as vibration insulators in bridges and tall buildings has direct relevance to modern methods of earthquake protection [40]. The industrial application of such components is wide spread, including engine mounts, off-shore structure, flex-joints and vibration insulators. A particular application, which in some extent motivated recent studies, is the increasing use of rubber-like components (slightly compressible). It should be emphasized that although surface waves are perhaps most readily associated with earthquakes, they are often observed as a significant contribution to the transient response of plates and laminates to impulsive loads [31]. Since the nature avoids any extremes, the slight compressibility seems to be a more realistic assumption than ideal incompressibility. Keeping these things in mind, the slightly compressible medium has been considered on the present study. Some remarkable works on this topic have been done by Dowaikh and Ogden [23], Rogerson and Fu [32], Ogden and Sotiropoulos [29], Rogerson and Sandiford [34], Rogerson and Murphy [33], Sandiford and Rogerson [37], Chattopadhyay and Sahu [18], Rogerson et al. [35], and Dhua et al. [22].

Results of theoretical and experimental studies revealed that a real earth is considerably more complicated than the models presented earlier. Therefore, the study of moving load in elastic media with irregular boundary [15] plays an important role for better understanding the behavior of stresses generated due to moving load at continental margins, mountain basins and roots, salt and ore bodies. Because the analytical treatment of the irregularities of the surface, in general, entails formidable mathematical difficulties, most of the researchers concentrated their effort with considerable success in considering the case of slightly curved surfaces of different geometries [16,27,38].

Deformation is the nonlinear behavior of materials, partly due to the characteristic of the material [3]. To obtain analytical solution to a problem involving finite deformation of elastic solid it is often necessary to adopt a specific form for the strain-energy function. Fundamental solutions in homogeneously deformed media were proposed and applied into an integral formulation by Bigoni and Capuani [4] and Brun et al. [11] for static problems and by Bigoni and Capuani [5] for dynamic problems. The case of surface instability

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http://dx.doi.org/10.1016/j.mechrescom.2015.03.012
0093-6413/

of an elastic anisotropic half space may occur under initial stress in finite strain [7]. Also, the surface instability cannot exist in the absence of a free surface. Internal buckling is a type of instability that may occur in a homogeneous medium of infinite extent under initial stress [6]. Some recent works considering initially stressed media have been done by Chatterjee et al. [14,13]. The stability analysis of the elastic half space is restricted to the case of a medium which is isotropic for incremental plane strain [9]. In particular, this property has been applied for rubber-type elasticity. Biot [8] has used the stability analysis in derivation of exact analysis for the surface instability of rubber in finite strain.

From this point of view, the mechanism of an earthquake is represented by a shear fracture produced by the drop in stress in the focal region. Fracture initiates at a point of the fault when the stress acting on the fault plane exceeds a critical value, propagates with a certain velocity, and finally stops when conditions impede its further propagation.

The response of moving load over a surface is a subject of continued interest due to its possible practical applications in determining the strength of a structure. The steady state solution of the problem of moving load over an elastic half space was investigated by Cole and Huth [20]. Craggs [21] established a relatively simple closed-form solution, exhibiting a resonance effect at a critical load velocity, which is equal to the velocity of Rayleigh wave. The problem considered by Cole and Huth [20] was discussed previously by Sneddon [42] by a somewhat different method. The problem of a moving load on a plate resting on an elastic half space has been solved by Sackman [36] and Miles [25]. The relevant systematic approach toward problem of moving load on a plate resting on an elastic half space has been given by Achenbach et al. [1], Chonan [19], Ungar [43], Olsson [30], Lee and Ng [24] and Alkeseyeva [2]. Stresses developed in a transversely isotropic elastic half space due to normal moving load over a rough surface have been determined by Mukherjee [26]. Brock [10] has studied the rapid sliding contact on a highly elastic pre-stressed material. Chattopadhyay and Saha [17] have studied the dynamic response of a normal moving load in the plane of symmetry of a monoclinic half space.

But most of the above authors have not studied the stresses developed in an irregular slightly compressible half space due to a moving normal load at the rough surface. Due to moving load in a half space, stresses will be developed. If the shear stress developed in the isotropic medium is larger than the strength of the medium, the failure occurs. Thus, the determination of stresses developed due to a moving load is an important task in this process.

Because of its closeness to the natural situation, the study of the stresses developed in a medium due to a moving load has gained great importance in mechanics. Due to a normal load moving on an irregular surface of a slightly compressible, finitely deformed half space, stresses will be developed. By virtue of stresses developed, failure in the medium can be developed. This can be calculated by knowing the strength of the medium, which is a function of strength parameters. Thus, the determination of stresses developed due to a moving load is an important task in this process. In the present paper, the stresses have been obtained in closed-form and then computed numerically for $R = 0.002$, 0.02 and 0.2, where R is the frictional coefficient of the irregular rough surface of a slightly compressible, finitely deformed half space. Finally, different graphs for different parameters have been plotted and discussed.

2. The governing equations

Consider an elastic solid possessing a natural unstressed isotropic state B_0 in a configuration for which the position vector of a representative particle is denoted by X_A. An initial primary deformation is imposed on the unstressed state to arrive at a finitely deformed equilibrium configuration, denoted by B_e. This particle has position vector $x_i(X_A)$. Finally, an infinitesimal time-dependent motion is super-imposed upon the finite deformation $B_0 \to B_e$ with the associated position vector of a representative particle in the current configuration B_t denoted by $\bar{x}_i(X_A, t)$. The position vector $\bar{x}_i(X_A, t)$ may therefore be expressed as

$$\bar{x}_i(X_A, t) = x_i(X_A) + u(X_A, t) \tag{1}$$

where u is the small time-dependent displacement associated with the secondary deformation $B_e \to B_t$. The deformation gradients associated with the deformations $B_0 \to B_t$ and $B_0 \to B_e$ are defined through the component relation

$$F_{iA} = \frac{\partial \bar{x}_i}{\partial X_A}, \quad \bar{F}_{iA} = \frac{\partial x_i}{\partial X_A} \tag{2}$$

respectively. Using Eqs. (1) and (2) it can be shown that the above two deformation gradients are related by

$$F_{iA} = (\delta_{ij} + u_{i,j})\bar{F}_{jA}, \tag{3}$$

with δ_{ij} the Kronecker delta, a comma an indication of differentiation with respect to the implied spatial coordinate in B_e and an overbar denoting evaluation in B_e.

It will be assumed that the pre-stress arises from a pure homogeneous strain and therefore $\bar{F}$ is a constant tensor field. In the absence of body forces, the equations of motion may be written as

$$(\pi_{iA}\bar{F}_{pA})_{,p} = \rho \ddot{u}_i, \quad \pi_{iA} = \frac{\partial W}{\partial F_{iA}} \tag{4}$$

with π_{iA} denoting the component of the first Piola–Kirchhoff stress tensor, ρ the mass density, a superimposed dot differentiation with respect to time and $W(\bar{F})$ is the strain energy function per unit volume.

Throughout this paper, our concern is nearly incompressible elastic materials. It can be shown for materials subject to the internal constraint of incompressibility that $J = 1$, where $J = \det F$, see e.g. [12]. This constraint is relaxed slightly to model such cases such that $J \approx 1$. Accordingly, an appropriate form of the strain energy function may now be obtained by introducing a Taylor series expansion of the strain energy function around the small order parameter $(J - 1)$, namely

$$W(F, J) = W_0(F, 1) + \left\{ \frac{\kappa (J-1)^2}{2} \right\}, \quad \kappa = \frac{\partial^2 W}{\partial J^2}. \tag{5}$$

Such a form of strain function has previously employed to elucidate anomalies on the slowness surface of incompressible and nearly incompressible elastic material [39]. It is noted that there is no linear term in the expansion of (5). This is because $\kappa \sim O(J-1)^{-1}$ and, therefore, nothing is gained by including a linear term [39]. In the above expansion (5), κ is the bulk modulus, which is evaluated at $J = 1$. Strictly speaking, the bulk modulus is a function of the deformation, however, for rubber-like materials it is found adequate to take the bulk modulus as constant [28].

In the absence of body forces, the linearized equations governing small-amplitude motion in an elastic material, subjected to a state of homogeneous finite deformation, are given by

$$B_{jilk} u_{k,lj} + \kappa \bar{J} u_{k,ki} = \rho \ddot{u}_i, \tag{6}$$

where B is the fourth-order elasticity tensor defined in component form as

$$B_{jilk} = \bar{J}^{-1} \bar{F}_{jA} \bar{F}_{lC} \left. \frac{\partial^2 W(F, 1)}{\partial \bar{F}_{iA} \bar{F}_{kC}} \right|_{F=\bar{F}}. \tag{7}$$

The elasticity tensor associated with a strain energy function of the form in Eq. (4) is derivable from Eq. (3), taking the component