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Analysis of a cracked concrete containing an inclusion with inhomogeneously imperfect interface

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ABSTRACT

The distributed dislocation technique is applied to determine the behavior of a cracked concrete matrix containing an inclusion. The analysis of cracked concrete in the presence of inclusions such as steel expansions is a practical problem that needs special attention. The solution to the problem of interaction of an edge dislocation with a circular inclusion having circumferentially inhomogeneously imperfect interface is available in the literature. This analytical solution is used in the distributed dislocation technique to obtain the stress intensity factor for the cracked concrete in the presence of inclusion. The interface of the matrix and the inclusion is assumed inhomogeneously imperfect and the stress intensity factor is determined for the cracked concrete for a case of two identical cracks on diametrically opposite sides of the inclusion. Consideration of this general inhomogeneously imperfect interface is the contribution of this paper. The variation of the inhomogeneity parameters is studied and presented. Additionally, the general assumption for the interface is simplified to the special case of perfectly bonded interface. The observations for the perfect interface are coincident with the previously reported results.

1. Introduction

Cracks can appear in the interface between concrete and steel expansion, which results in negative implications in building performance [3]. Such inclusion-matrix system also occurs in fiber-reinforced composites (FRCs).

The analysis of interaction of cracked concrete and inclusion with perfectly bonded interface was carried out by Leung [9]. Rabczuk et al. [16] presented a three-dimensional mesh-free method for modeling arbitrary crack initiation and crack growth in reinforced concrete structure and compared the results to experimental data. Mantic [11] investigated crack onset at the interface between a stiff circular cylindrical inclusion and a compliant unbounded matrix subjected to a remote uniaxial transverse tension. Later, Sosa [17] studied the nonlinear interface in reinforced concrete for modeling the pull-out problems and reported comparison with available experimental data. Liu et al. [10] simulated the influence of inclusion size and strain rate on the interaction between cracking and the matrix-inclusion interface in composites.

Dislocations are building blocks of cracks. In dislocation-based fracture mechanics [21], it has been proved that a crack can be considered as a composed defect created by a continuous distribution of dislocations. In this approach, the study of the interaction of dislocations with interfaces is required to investigate the interaction of cracked concrete and inclusion. The interaction of a dislocation and the inclusion has been the topic of some of the investigations, while having different assumption as the condition of interface.

The interface between the inclusion and the matrix may be modeled in various forms. Dundurs and Mura [4] assumed the perfectly bonded interface and analyzed the interaction between an edge dislocation and a circular inclusion. Later, Newman [15] considered the perfectly smoothed interface with free sliding and reported the stress intensity factor for a cracked matrix in the presence of an inclusion. The problem of a three-phase circular inclusion interacting with a dislocation in antiplane shear was presented by Xiao and Chen [22], while the analysis was carried out for the case of the classical perfect bonding condition. Later, Wang and Shen [20] solved the problem of an edge dislocation in a three-phase composite cylinder model with the assumption of sliding interface. In 2003, Sudak [18] obtained the solution for a homogeneous circular inclusion interacting with a dislocation under thermal loadings in antiplane shear. The bonding along the inhomogeneity–matrix interface was considered to be imperfect with the assumption that the interface imperfections were constant.

Imperfect interface has been the topic of some recent investigations. Fan and Wang [6] applied a linear spring model to study imperfect interfaces. They formulated the interaction of a screw dislocation with an imperfect interface and the results showed

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that the interacting force on a screw dislocation with an imperfect interface vary between that with a free surface and that with a perfect interface. Later, Wang [19] considered a more general case and studied the interaction problem of an edge dislocation with a circular inclusion with a circumferentially inhomogeneously imperfect interface. He modeled the interface as a spring (interphase) layer with vanishing thickness, having a displacement jump across the interface in the same direction as the corresponding tractions, while the interface parameter was non-uniform along the interface.

Interaction of an edge dislocation with a coated elliptic inclusion was also studied by Chen et al. [2]. They obtained the general expressions of the displacements and stresses, where an edge dislocation is located in matrix, coating layer and inclusion. Recently, Zhang et al. [23] presented a general and approximate solution for an edge dislocation interacting with an inhomogeneity of arbitrary shape under combined dislocation and applied stress fields. As a special case, explicit solution for the interaction of an edge dislocation with a small circular inhomogeneity was studied.

The above-mentioned analytical solutions to the dislocation are very useful in the study of fracture behavior of materials. These solutions may be utilized in the distributed dislocation technique to determine the fracture parameters such as stress intensity factor ([8,21]). This technique has been used for the analysis of cracked structures in static loading ([5,7]) and elastodynamic loading [12]. Smart materials such as piezoelectric and piezomagnetic are also investigated by means of this method [13,14]. Leung [9] applied this technique for the analysis of interaction of cracked concrete and inclusion with perfectly bonded interface. His approach is based on the dislocation solution presented by Dundurs and Mura [4].

In the present study, the solution of the interaction of an edge dislocation and a circular inclusion with an inhomogeneously imperfect interface (reported by Wang [19]), is applied in the technique of the distributed dislocation to study the interaction of cracked concrete and inclusion with inhomogeneously imperfect interface. The results are simplified for the special case of perfectly bonded interface and have been verified with the available results. The novelty of this paper is the generalization of the perfect interface assumption to imperfect interface for the analysis of inclusions in the cracked concrete.

This paper is organized as follows. In Section 2, we review the solution to the problem of interaction between a screw dislocation and a circular inclusion. Then, in Section 3, the distributed dislocation technique is applied to model the cracked concrete containing an inclusion. In the next section, the results are verified for perfect interface and the results are presented for imperfect interface. The conclusion is provided in Section 5.

2. Interaction between dislocation and a circular inclusion

In order to analyze the behavior of the cracked concrete in the presence of an inclusion, the distributed dislocation technique is applied. To do so, the solution of the problem of the dislocation in a matrix containing an inclusion is required. As mentioned in the previous section, this analysis might be carried out with different interface conditions. The inclusion with an inhomogeneously imperfect interface is a general case which can be simplified to the simple case of perfectly bonded interface. The solution of the interaction of an edge dislocation and a circular inclusion with an inhomogeneously imperfect interface has been reported by Wang [19].

Consider a circular inclusion S_1 of radius R embedded in an unbounded matrix S_2 (Fig. 1). The subscripts 1 and 2 will refer to the regions S_1 and S_2, respectively. A straight climb edge dislocation with Burgers vector b_y is located on x-axis at $x = x_0$ in the matrix.

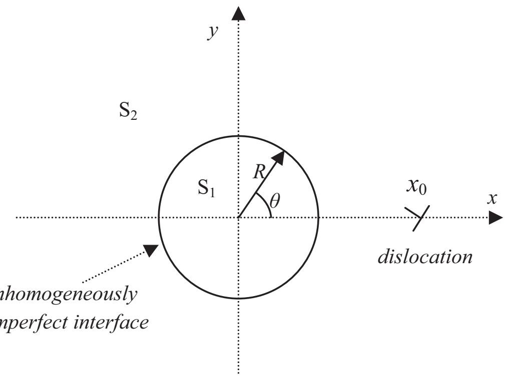

Fig. 1. An edge dislocation interacting with a circular inclusion with an inhomogeneously imperfect interface.

The imperfect interface is assumed to have the following inhomogeneous boundary conditions:

$$\begin{cases} \sigma_{rr}^{(1)} = \sigma_{rr}^{(2)} = \eta(\theta)\,\|u_r\|\,, \\ \sigma_{r\theta}^{(1)} = \sigma_{r\theta}^{(2)} = \eta(\theta)\,\|u_\theta\|\,, \end{cases} \qquad |z| = R \tag{1}$$

where $z = x + iy = re^{i\theta}$ is the complex coordinate, $\|*\| = (*)_2 - (*)_1$ denotes the jump across the interface and $\eta(\theta)$ is a nonnegative periodic function of the angle θ. $\sigma_{\alpha\beta}$ ($\alpha,\beta \in \{r,\theta\}$) depicts the stress components and the superscript (1) indicates the quantities pertaining to the inclusion while the superscript (2) indicates the quantities pertaining to the matrix.

Physically, Eq. (1) implies that the same degree of imperfection is realized in the normal and tangential directions, and that the displacement (u_r, u_θ) jump across the interface is in the same direction as the corresponding tractions. The non-uniform interface parameter $\eta(\theta)$ is selected as

$$\frac{1}{\eta(\theta)} = a_0 + a_1 e^{i\theta} + \bar{a}_1 e^{-i\theta} \tag{2}$$

where a_0 and a_1 are the interface parameters such that $a_0 \geq 2|a_1|$ to ensure the non-negative value of $\eta(\theta)$.

Having the boundary conditions (1), the stress field induced by the dislocation in the inclusion and in the matrix (Fig. 1) is expressed in terms of the complex potentials, $\phi_k(\zeta)$ and $\psi_k(\zeta)$, as follows [19]

$$\frac{\sigma_{yy}^{(k)}(x, x_0)}{b_y} = Re\left\{ 2\frac{\phi_k(\zeta)}{m'(\zeta)} + \overline{m'(\zeta)}\left\{\frac{\phi_k{}'(\zeta)}{m'(\zeta)}\right\}' + \frac{\psi_k{}'(\zeta)}{m'(\zeta)} \right\}, \quad k = 1, 2 \tag{3}$$

where prime denotes differentiation with respect to ζ, bar-notation represents the complex conjugate, and the complex potentials, $\phi_k(\zeta)$, $\psi_k(\zeta)$ are introduced in the appendix A. $m(\zeta)$ is a conformal mapping function

$$z = m(\zeta) = (\zeta - \rho) / \left(\frac{\bar{\rho}}{R^2}\zeta - 1 \right) \tag{4}$$

where $\rho = R\left(-a_0 + \sqrt{a_0{}^2 - 4|a_1|^2}\right)/(2a_1)$, $\left(|\rho| < R\right)$. Using this solution, the distributed dislocation technique can be applied to the analysis of cracked matrix in presence of inclusion.

3. Crack modeling

The distributed dislocation technique is a popular tool for the analysis of cracked structures [8,21]. To apply this technique for a structure, an analytical solution for the problem of the dislocation in the same structure is required. Then, by continuously distributing the dislocations along the crack lines, the cracked structure is