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Opening and contact zones of an interface crack in a piezoelectric bimaterial under combined compressive-shear loading

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ABSTRACT

A plane problem for a tunnel electrically permeable interface crack between two semi-infinite piezoelectric spaces is studied. A remote mechanical and electrical loading is applied. Elastic displacements and potential jumps as well as stresses and electrical displacement along the interface are presented using a sectionally holomorphic vector function. It is assumed that the interface crack includes zones of crack opening and frictionless contact. The problem is reduced to a combined Dirichlet–Riemann boundary value problem which is solved analytically. From the obtained solution, simple analytical expressions are derived for all mechanical and electrical characteristics at the interface. A quite simple transcendental equation, which determines the point of separation of open and close sections of the crack, is found. For the analysis of the obtained results, the main attention is devoted to the case of compressive-shear loading. The analytical analysis and numerical results show that, even if the applied normal stress is compressive, a certain crack opening zone exists for all considered loading values provided the shear field is present. It is found that the shear stress intensity factor at the closed crack tip and the energy release rates at the both crack tips depend very slightly on the magnitude of compressive loading.

1. Introduction

Piezoelectric materials are active materials with an intrinsic electro-mechanical coupling. They are widely used in engineering as sensors, transducers and actuators. Appearance of interface cracks in such devices and their growth is an important limiting factor for their load carrying capacity and functionality. Because of the practical and fundamental importance of the problem, interface cracks in piezoelectric materials have been actively studied in the last several decades. A traction-free and electrically permeable crack between a piezoelectric material and a conductor was considered by Kudryavtsev et al. [14]. Another particular case of electrical conditions on the crack faces has been examined by Suo et al. [25]. An interface crack was assumed to be electrically insulated and a new type of singularity occurring for such electrical conditions has been found in this paper. The above-mentioned studies were performed within the framework of the classical interface crack model [26]. This model assumes that the crack is completely open and usually leads to oscillating singularity at the crack tips and physically unreal overlapping of the crack faces. To eliminate this phenomenon, a contact zone model for a crack between isotropic materials was suggested by Comninou [3] and further developed by Atkinson [9], Simonov [24], Dundurs and Gautesen [6] and others. The contact zone approach was applied to interface cracks in piezoelectric materials by Qin and Mai [22] with the assumption of electrical insulation of the contact zone and by Herrmann and Loboda [10] and Herrmann et al. [11] for an electrically permeable and electrically impermeable cracks, respectively. A periodic set of limited permeable interface cracks with contact zones in a piezoelectric bimaterial was considered by Kozinov et al. [13]. An electrically conducting interface crack with a contact zone in a piezoelectric bimaterial was studied by Loboda et al. [18]. The extended Green's functions for an extended dislocation and displacement discontinuity located at the interface of a piezoelectric bimaterial were obtained by Zhao et al. [27]. Different variants of electric and magnetic permeability of interface cracks with contact zones in piezoelectric/piezomagnetic bimaterials were investigated by Herrmann et al. [12], Feng et al. [7] and Ma et al. [19].

The validity of a simplified electric boundary condition at crack faces in a homogeneous piezoelectric material has been investigated by many authors. Gao and Fan [8] considered a slit crack as a limiting case of an elliptical hole or an inclusion. Taking into

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account the exact electric field in the mentioned hole or inclusion, the authors arrived at the conclusion that the assumption of a permeable crack is more realistic than that of an impermeable one.

The most investigations related to an interface crack were performed for tensile or tensile-shear loading acting outside of the crack. Such loading is the most dangerous concerning the possibility of the cracked materials fracture. However sometimes piezoelectric composite materials with interface defects work in a compressed or a compressed-shear field. Under such loading, the cracks in homogeneous materials are usually close, but interface cracks can be partially open in the presence of additional shear loading. For isotropic bimaterials this possibility was shown by Comninou and Schmueser [4] with use of singular integral equations and by Atkinson [1], Simonov [24] and Gautesen and Dundurs [9] in an analytical way. Related problems for isotropic Brazilian disk specimen were studied in a numerical way by Atkinson et al. [2] and Dorogoy and Banks-Sills [5]. However to the authors' best knowledge the similar problem for piezoelectric materials has not been considered yet.

In the present paper, a plane problem for a tunnel electrically permeable crack between two piezoelectric spaces is considered. The main attention is devoted to the case of compressive-shear loading. Zones of the crack opening and frictionless contact are assumed to arise along the crack area. The existence of such zones is confirmed by exact analytical solution of the problem. Their lengths are found in an analytical form with respect to material combinations and external loadings.

2. General solution of the basic equation

The constitutive relations for a linear piezoelectric material in the absence of body forces and free charges can be presented in the form [21]

$$\sigma_{ij} = c_{ijkl}\gamma_{kl} - e_{kij}E_k, \tag{1}$$

$$D_i = e_{ikl}\gamma_{kl} + \varepsilon_{ik}E_k, \tag{2}$$

$$\sigma_{ij,i} = 0, \quad D_{i,i} = 0, \tag{3}$$

$$\gamma_{ij} = 0.5(u_{i,j} + u_{j,i}), \quad E_i = -\varphi_i, \tag{4}$$

where u_k, φ, σ_{ij}, γ_{ij} and D_i are the elastic displacements, electric potential, stresses, strains and electric displacements, respectively; c_{ijkl}, e_{lij} and ε_{ij} are the elastic modules, piezoelectric constants and dielectric constants, respectively. The subscripts in (1)–(4) are ranging from 1 to 3 and Einstein's summation convention is used in (1)–(3).

In the papers by Herrmann and Loboda [10] similarly to the solution by Suo et al. [25], the following representations have been derived for a piezoelectric bimaterial plane

$$\left\langle \mathbf{V}'(x_1, 0)\right\rangle = \mathbf{W}^+(x_1) - \mathbf{W}^-(x_1), \tag{5}$$

$$\mathbf{t}^{(1)}(x_1, 0) = \mathbf{G}\mathbf{W}^+(x_1) - \bar{\mathbf{G}}\mathbf{W}^-(x_1), \tag{6}$$

where

$$\left\langle \mathbf{V}'(x_1, 0)\right\rangle = \mathbf{V}'^{(1)}(x_1, 0) - \mathbf{V}'^{(2)}(x_1, 0), \tag{7}$$

and $\mathbf{G} = \mathbf{B}^{(1)}\mathbf{D}^{-1}$, $\mathbf{D} = \mathbf{A}^{(1)} - \bar{\mathbf{A}}^{(2)}\left(\bar{\mathbf{B}}^{(2)}\right)^{-1}\mathbf{B}^{(1)}$, $\mathbf{W}^+(x_1) = \mathbf{W}(x_1 + i0)$, $\mathbf{W}^-(x_1) = \mathbf{W}(x_1 - i0)$; $\mathbf{A}^{(m)}$, $\mathbf{B}^{(m)}$ are known matrices [25] related to the upper (m=1) and lower (m=2) half-planes, respectively; $\mathbf{V} = [u_1, u_2, u_3, \phi]^T$ and $\mathbf{t} = [\sigma_{13}, \sigma_{23}, \sigma_{33}, D_3]^T$. It is worth to note that the unknown vector-function $\mathbf{W}(z) = [W_1(z), W_2(z), W_3(z), W_4(z)]^T$ is analytic in the whole plane including the bonded parts of the material interface ($z = x_1 + ix_3$, $i = \sqrt{-1}$). Moreover the $[4 \times 4]$ bimaterial matrix $\mathbf{G}$ and the vector-function $\mathbf{W}(z)$ are related to the matrix $\mathbf{H}$ and the vector function $\mathbf{h}(z)$ of the paper by Suo et al. [25] as $\mathrm{i}\mathbf{G}^{-1} = \mathbf{H}$, $\mathbf{W}(z) = -\mathrm{i}\mathbf{H}\mathbf{h}(z)$, respectively.

For the accepted kind of polarization the matrix $\mathbf{G}$ has the following structure

$$\mathbf{G} = \begin{bmatrix} ig_{11} & 0 & g_{13} & g_{14} \\ 0 & ig_{22} & 0 & 0 \\ g_{31} & 0 & ig_{33} & ig_{34} \\ g_{41} & 0 & ig_{43} & ig_{44} \end{bmatrix}, \tag{8}$$

where all g_{ij} are real and the relations $g_{31} = -g_{13}, g_{41} = -g_{14}, g_{43} = g_{34}$ hold true.

The analysis of the matrix (8) shows that the stress–strain state in this case can be decoupled into in-plane and out-of-plane problems. Because the out-of-plane problem is relatively simple the main attention will be devoted to the in-plane problem which is characterized by the displacements u_1, u_3 and the electric potential ϕ.

3. Formulation of the problem

A tunnel interface crack $c \le x_1 \le b$, $x_3 = 0$ between two semi-infinite ceramic spaces $x_3 > 0$ and $x_3 < 0$ which are poled in the x_3-direction is considered. The material properties are defined by the matrices $c_{ijkl}^{(1)}$, $e_{lij}^{(1)}$, $\varepsilon_{ij}^{(1)}$ (for $x_3 > 0$) and $c_{ijkl}^{(2)}$, $e_{lij}^{(2)}$, $\varepsilon_{ij}^{(2)}$ (for $x_3 < 0$). It is assumed that the direction of the polarization of both materials is orthogonal to the crack faces. The half-spaces are loaded at infinity with uniform stresses $\sigma_{33}^{(j)} = \sigma$, $\sigma_{13}^{(j)} = \tau$, $\sigma_{11}^{(j)} = \sigma_{xxj}^{\infty}$ and with uniform electric displacements $D_3^{(j)} = d$, $D_1^{(j)} = D_{xj}^{\infty}$ which satisfy the continuity conditions at the interface. Because the load does not depend on the coordinate x_2, the relations of the previous chapter can be used and a two-dimensional problem in (x_1, x_3)-plane can be considered (Fig. 1).

The main attention in this paper will be devoted to the case of shear and compressed normal loading. Nevertheless it will be assumed that the crack is open for $x_1 \in [c, a] = L_1$ whereas its faces are in frictionless contact for $x_1 \in (a, b) = L_2$, and the position of the point a is arbitrarily chosen for the time being.

The electric potential and the normal component of the electric displacement are assumed to be continuous across the whole interface, therefore the crack is assumed to be an electrically permeable one. According to Gao and Fan [8] such conditions are more realistic than the insulated one for many crack fillers and even for a crack filled by air (see [15] as well). Thus the continuity and boundary conditions at the interface are:

$$\text{For}\, x_1 \in L: \left\langle \mathbf{V}\right\rangle = 0, \quad \left\langle \mathbf{t}\right\rangle = 0, \tag{9}$$

$$\text{For}\, x_1 \in L_1: \sigma_{13}^{\pm} = 0, \quad \sigma_{33}^{\pm} = 0, \quad \left\langle \varphi\right\rangle = 0, \quad \left\langle D_3\right\rangle = 0, \tag{10}$$

$$\text{for}\, x_1 \in L_2: \langle u_3\rangle = 0, \quad \left\langle \varphi\right\rangle = 0, \quad \sigma_{13}^{\pm} = 0, \quad \langle \sigma_{33}\rangle = 0, \quad \left\langle D_3\right\rangle = 0. \tag{11}$$

4. Solution for an arbitrary position of the point "*a*"

Using the presentation (6), taking into account that $\left\langle \varphi\right\rangle = 0$ for $-\infty < x_1 < \infty$ and performing the analysis similar to [10], one arrives at the following equations

$$\sigma_{33}^{(1)}(x_1, 0) + im_j\sigma_{13}^{(1)}(x_1, 0) = t_j[F_j^+(x_1) + \gamma_j F_j^-(x_1)] + \sigma_0, \tag{12}$$

$$\left\langle u_1'(x_1, 0)\right\rangle + is_j\left\langle u_3'(x_1, 0)\right\rangle = F_j^+(x_1) - F_j^-(x_1), \tag{13}$$