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Off-center impact of an elastic column by a rigid mass

M. Rezvanian, A.M. Baghestani, M. Danesh Pazhooh, S.J. Fariborz*

Department of Mechanical Engineering, Amirkabir University of Technology (Tehran Polytechnic), 424, Hafez Avenue, Tehran, Iran

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ABSTRACT

Based on the Timoshenko beam model the equations of motion are obtained for large deflection of off-center impact of a column by a rigid mass via Hamilton's principle. These are a set of coupled nonlinear partial differential equations. The Newmark time integration scheme and differential quadrature method are employed to convert the equations into a set of nonlinear algebraic equations for displacement components. The equations are solved numerically and the effects of weight and velocity of the rigid mass and also off-center distance on deformation of the column are studied.

1. Introduction

The axial impact of a column at the center of cross-section by a rigid mass is an old problem, Davidson [1]. Several investigators analyzed the impact of beams with initial geometric imperfection, see e.g., Hayashi and Sano [2,3], Ari-Gur et al. [4], Ji and Waas [5]. The results of their analyses were dependent upon the amplitude of the initial imperfection. In the study of the impact of perfect columns conducted by Wang and Tian [6], and Ji and Waas [7], at the outset, a non-linear term in a differential equation of motion was ignored which led to the differential equation for linear wave propagation in axial direction. The solution to this equation was, then, used to investigate the transverse motion of the column. The nonlinear vibration of the Euler–Bernoulli beam under transverse periodic load subjected to axial impact was the subject of study by Awrejcewicz et al. [8]. In practice, however, the off-center impact of a straight column by a striking mass may occur, which leads to both transverse and axial deformation of the column. Contrary to the central impact of column by a mass, off-center impact has rarely been addressed in the literature. Kuo [9] experimentally investigated eccentric longitudinal impact of a horizontally suspended free beam by a striking bar which had a canonical end to exert the impact. A theoretical analysis of the response of the beam using Timoshenko beam theory was also carried out. The governing equations were a set of linear partial differential equations which was solved by the method of characteristics. The oblique impact at the free end of a cantilever column by a rigid mass was the subject of study by Ren [10]. The Timoshenko beam model was employed and the equations of motion were obtained employing Hamilton's principle. These were three non-linear partial differential equations which were solved by finite difference method. The dependencies of impact duration on the incidence angle and also on the slenderness ratio of a column were investigated. In another article, Ren and Kou [11] conducted an experiment to verify the results of their earlier analysis.

The off-center impact of a column by a rigid mass is the subject of the present study. The Timoshenko beam model is utilized. Therefore, the analysis is valid for moderately thick columns as well as slender ones. The application of Hamilton's principle results in a set of coupled non-linear differential equations for the displacement components of the column. These equations are solved numerically for an impacted column with simply supported or clamped condition at the fixed end. The results are in excellent agreement with those obtained by finite element method. The effects of velocity, off-center distance and weight of impacting mass on the deformation of column are studied.

2. Formulation

We consider a column with length l, thickness b, and height h. The dimensions are in the x-, y-, and z-directions, respectively. The column is simply supported at $x = l$; thus, at this end, transverse

* Corresponding author. Tel.: +98 2164543460.
E-mail addresses: rezvanian.m@gmail.com (M. Rezvanian), am_baghestany@yahoo.com (A.M. Baghestani), daneshm@aut.ac.ir (M.D. Pazhooh), sjfariborz@yahoo.com (S.J. Fariborz).

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displacement vanishes. The equations representing displacement field in the Timoshenko beams are

$$\begin{aligned} u_x &= u(x,t) + z\phi(x,t) \\ u_z &= w(x,t) \end{aligned} \tag{1}$$

where u and w are, respectively, the axial and transverse deformations of the beam axis, and ϕ, is the rotation of its cross-section. The stationary column is initially impacted by a rigid mass M with a velocity V_0 at the distance $z=e$ from the center of the cross-section. The kinetic energy of the system, in view of Eq. (1), becomes

$$\begin{aligned} T = {}& \frac{\rho b}{2}\int_0^l \int_{-h/2}^{h/2} \left[\left(\frac{\partial u}{\partial t} + z\frac{\partial \phi}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right] dxdz \\ & + \frac{M}{2}\left[\frac{\partial u}{\partial t}(l,t) + e\frac{\partial \phi}{\partial t}(l,t)\right]^2 \end{aligned} \tag{2}$$

where ρ is the mass density of the column. The potential energy, under the hypothesis of elastic impact, yields

$$U = \frac{b}{2}\int_0^l \int_{-h/2}^{h/2} (\sigma_{xx}\varepsilon_{xx} + \kappa\sigma_{xz}\gamma_{xz})dxdz - Mgu(l,t) - Mge\phi(l,t) \tag{3}$$

where $\kappa = 5/6$ is the correction shear factor. For isotropic beams with infinitesimal strains the constitutive equations obey Hooke's; thus, $\sigma_{xx} = E\varepsilon_{xx}$, $\sigma_{xz} = G\gamma_{xz}$, where, E and G are Young's and shear moduli, respectively. The von-Karman strain measure for a Timoshenko beam results in

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} + z\frac{\partial \phi}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \gamma_{xz} &= \frac{\partial w}{\partial x} + \phi \end{aligned} \tag{4}$$

Therefore, potential energy (3) in view of Eq. (4), becomes

$$\begin{aligned} U = {}& \frac{b}{2}\int_0^l \int_{-h/2}^{h/2} \left\{ E\left[\frac{\partial u}{\partial x} + z\frac{\partial \phi}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]^2 \right. \\ & \left. + G\kappa\left[\frac{\partial w}{\partial x} + \phi\right]^2 \right\} dxdz - Mgu(l,t) - Mge\phi(l,t) \end{aligned} \tag{5}$$

Hamilton's principle states that

$$\int_0^{t_0} \delta(T-U)dt = 0 \tag{6}$$

where $[0, t_0]$ is a small but known time interval. Substituting Eqs. (2) and (5) into Eq. (6) and carrying out the required manipulation leads to the following equations of motion

$$\begin{aligned} & \frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 u}{\partial t^2} \\ & \frac{\kappa}{2(1+\nu)}\left(\frac{\partial \phi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + \frac{\partial^2 u}{\partial x^2}\frac{\partial w}{\partial x} + \frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2} \\ & \quad + \frac{3}{2}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2 = \frac{1}{C^2}\frac{\partial^2 w}{\partial t^2} \\ & \frac{\partial^2 \phi}{\partial x^2} - \frac{6\kappa}{(1+\nu)h^2}\left(\frac{\partial w}{\partial x} + \phi\right) = \frac{1}{C^2}\frac{\partial^2 \phi}{\partial t^2} \end{aligned} \tag{7}$$

where ν is the Poisson's ratio and $C = \sqrt{E/\rho}$ is the longitudinal wave velocity in the material. The above equations are subjected to the following natural and geometric boundary conditions at, $x=l$

$$\begin{aligned} & \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{M}{Ebh}\left(g - \frac{\partial^2 u}{\partial t^2} - e\frac{\partial^2 \phi}{\partial t^2}\right) = 0 \\ & \frac{\partial \phi}{\partial x} - \frac{12Me}{Ebh^3}\left(g - \frac{\partial^2 u}{\partial t^2} - e\frac{\partial^2 \phi}{\partial t^2}\right) = 0, \quad w = 0 \end{aligned} \tag{8}$$

And at, $x=0$

$$u = 0 \quad \text{and} \quad w = 0, \tag{9}$$

The last boundary condition at $x=0$ for a column with simply supported and clamped boundaries are $(\partial\phi/\partial x) = 0$, and $\phi = 0$, respectively. Let the impacted column be stationary at $t=0$, the initial conditions for Eq. (7), yield

$$\begin{aligned} & u(x,0) = \phi(x,0) = w(x,0) = 0 \\ & \frac{\partial \phi}{\partial t}(x,0) = \frac{\partial w}{\partial t}(x,0) = 0, \quad x \in [0,l] \\ & \frac{\partial u}{\partial t}(x,0) = 0, \quad x \in [0,l) \\ & \frac{\partial u}{\partial t}(l,0) = -V_0 \end{aligned} \tag{10}$$

Furthermore, as the axial force at $x=l$ vanishes, the impacting mass losses contact with the column. From first Eq. (4) and Hooke's law, we arrive at the equation of the contact duration, t_c, as

$$\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]_{x=l} = 0 \tag{11}$$

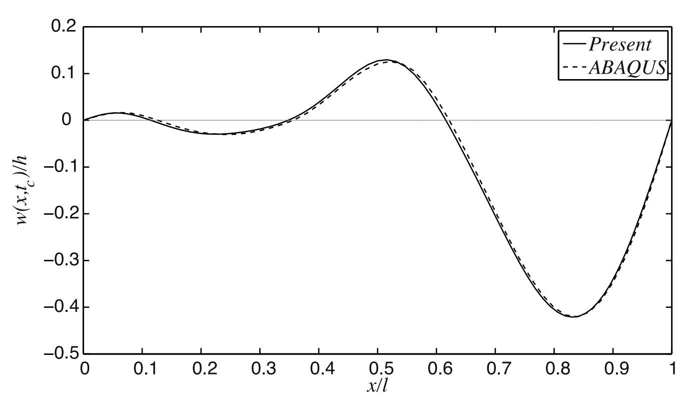

Fig. 1. Transverse deflection of the column simply supported at $x=0$.

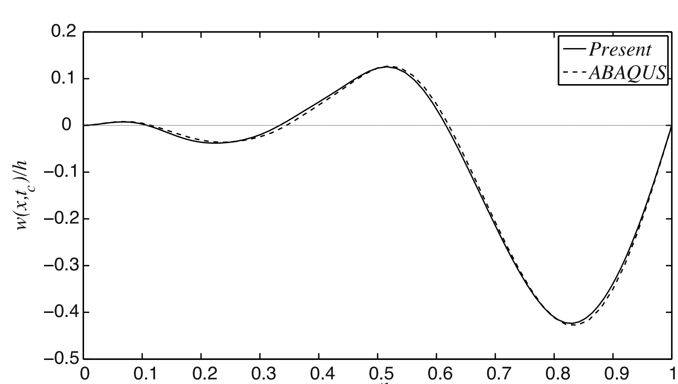

Fig. 2. Transverse deflection of the column clamped at $x=0$.