Mechanics Research Communications 63 (2015) 48–53

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

An asymptotic solution for fluid production from an elliptical hydraulic fracture at early-times

Yan Jin[a], Kang Ping Chen[b,*], Mian Chen[a]

[a] College of Petroleum Engineering, China University of Petroleum (Beijing), Beijing 102200, China
[b] School for Engineering of Matter, Transport and Energy, Arizona State University, Tempe, AZ 85287-6106, USA

ARTICLE INFO

Article history:
Received 30 July 2014
Received in revised form 8 December 2014
Accepted 16 December 2014
Available online 25 December 2014

Keywords:
Asymptotic solution
Transient pressure
Hydraulic fractured wells

ABSTRACT

The early-time transient flow during the start-up of fluid production from a porous medium by a well intersected by a vertical elliptical hydraulic fracture is studied using an asymptotic analysis. The analysis is focused on the situation of practical interest where the fracture conductivity is high so that production from the fracture dominates. The first three terms in a short-time asymptotic expansion for the production rate during constant-pressure production, and for the well-pressure during constant-rate production, are obtained. It is shown that the fracture tip starts to influence the production rate only when the dimensionless time is increased to the square of the reciprocal of the dimensionless fracture conductivity. The asymptotic results also show that geometric factors of an elliptical fracture introduce non-negligible corrections to the so-called bilinear flow in the early times, which were previously erroneously associated with the effect of the fracture tip.

1. Introduction

Hydraulic fracturing is the key technology behind the recent spectacular success in shale gas production in North America. The effectiveness of a hydraulically fractured well is commonly evaluated by a pre-production pressure transient analysis which involves two elements: the first part is to propose a model for the flows in the reservoir and the fracture, and to solve the corresponding governing equations for the pressure fields; the second part is to measure the well pressure and the flow-rate in the field and to match these data to those predicted from the proposed model. The reservoir and fracture properties extracted from this analysis are then used to assess the success of the hydraulic fracturing via a production rate decline analysis. There is a voluminous literature on predicting transient pressure response of hydraulically fractured wells for various models which consider different aspects of the problem [1–12]. For example, one of the best-known models is the finite conductivity fracture model developed by Cinco-Ley and collaborators [5,7]. Cinco-Ley et al. [5] numerically solved an integral equation to obtain the well pressure response for constant-rate production; while Cinco-Ley and Samaniego [7] analyzed a simplified model and identified various flow regimes. It is particularly worth noting that in the pre-production pressure transient analysis, analytical and semi-analytical (i.e. analytical in the Laplace transform space) solutions are preferred, as the slope of a solution is often identified with a flow-regime and used to extract property values [7,10]. While most of the works have modeled the fracture as a rectangular slit [13,1,15,14,23,9,11,16], among others, have studied the behavior of elliptical fractures. A primary reason of using a nearly degenerate ellipse to represent a thin, long fracture is that elliptic coordinates can be used to solve the pressure field. This overcomes the difficulties, such as singularity and slow convergence, encountered in solving partial differential equations with a mixed boundary condition in Cartesian coordinates [9]. While Prats et al. [1] and Kuchuk and Brigham [15] studied the special case of infinite fracture conductivity, Riley [9] carried out investigations on the transient pressure behavior of an elliptical fracture with a finite conductivity, focusing on the exact analytical solution in the Laplace transform space. It is noted that Riley's work has been used as the benchmark for many recent developments on elliptical flows (e.g. [11,16]). Riley's transient pressure solution involves Mathius functions which are cumbersome to evaluate, and they are particularly difficult to compute for very short times relative to the production time scale. For such early-times, Riley also developed approximate composite solutions in the Laplace transform space, which are made of two different composite solutions obtained from an *ad hoc* combination of three solutions for three simplified cases: a solution for an infinitely long fracture with a rectangular cross section [8]; a solution for an elliptical fracture with a one-dimensional reservoir flow; and a solution for an infinite conductivity fracture.

* Corresponding author. Tel.: +1 480 965 0907; fax: +1 480 727 9321.
E-mail address: k.p.chen@asu.edu (K.P. Chen).

http://dx.doi.org/10.1016/j.mechrescom.2014.12.004
0093-6413/

Riley [9] further assigned one of his two composite solutions to the dimensionless fracture conductivity range of $C_{fD} \leq 45$ and the other to $C_{fD} > 45$ without delineating the reason why $C_{fD} = 45$ plays such an important role in his short-time solutions. He also suggested the use of these approximate well pressure solutions for dimensionless times less than 10^{-3}, apparently based on his observation that elliptical effect can persist even at such small times. No evidence of such an effect, however, was presented by Riley [9]. Riley's solutions, whether in terms of the infinite series of Mathius' and cosine functions, or the approximations in the short-time limit, are analytical in the Laplace transform space only. Numerical inversions are still required to convert them into solutions in the physical space.

The purpose of the present paper is to seek a short-time analytical solution in the *physical space* for the fluid production rate from vertically fractured wells using consistent asymptotic expansions. "Short time" is here defined as the initial short period relative to the production time scale; the latter is usually very long, in the order of months or years. The hydraulic fracture is modeled geometrically as a nearly degenerate ellipse. The study is focused on the range of the dimensionless fracture conductivity for which fluid production comes almost entirely from the fracture and the production from the wellbore can be neglected. The asymptotic solutions sought are valid for both finite and unbounded reservoirs, since within short times after the instantaneous start-up of production only fluid in the close vicinity of the fracture begin to deplete its pressure, and the far boundary of a finite reservoir acts as if it were located at infinity. Asymptotic solutions for the pressure in the reservoir and in the fracture are obtained in the Laplace transform space; and they are used to derive the first three terms in an asymptotic expansion for the production rate in the case of constant-pressure production; and for the well pressure in the case of constant-rate production. These asymptotic solutions are valid when the product $C_{fD}t^{1/2}$ is small, where t is the dimensionless time measured in terms of the production time scale. When $C_{fD}t^{1/2}$ is about one, the fracture tip effect, which is necessary for the development of elliptical flows, starts to come into play. This critical value for the product $C_{fD}t^{1/2}$ reflects the underlying physics of the start-up process: the fracture tip effect is felt within a short time of the start-up for high conductivity fractures; while it takes a longer time for this effect to manifest for low conductivity fractures. The incipient time for the tip effect to influence the fluid production is shown to scale with the inverse square of the fracture conductivity C_{fD}^2. The asymptotic results obtained in this study also show that Riley's perception that the elliptical effect can persist even at small times is entirely erroneous.

2. Mathematical formulation and asymptotic solutions for pressure in short times

Consider fluid production from a fully penetrated vertically-fractured well (Fig. 1). The hydraulic fracture is modeled as a thin and long ellipse intersecting the wellbore with a fracture width w_f, which is much smaller than the wellbore diameter. The Cartesian coordinates (x,y) and the elliptic coordinates (ξ,η) are related by $x = L\cos h\xi \cos\eta$, $y = L\sin h\xi \sin\eta$, with L being essentially the fracture half-length, and $L \gg w_f$. *The hydraulic fracture is supported by proppants and it can be considered as incompressible*. Subscripts "m", "f", are used for reservoir and fracture quantities, respectively. The permeabilities in the reservoir and the hydraulic fracture are κ_m, κ_f, respectively, with $\kappa_f \gg \kappa_m$. *For any successful hydraulic fracturing job*, the dimensionless fracture conductivity $C_{fD} = \kappa_f w_f/(\kappa_m L) > 10$. Thus, the fluid production is nearly entirely from the fracture, and the contribution from the wellbore to the production is negligible.

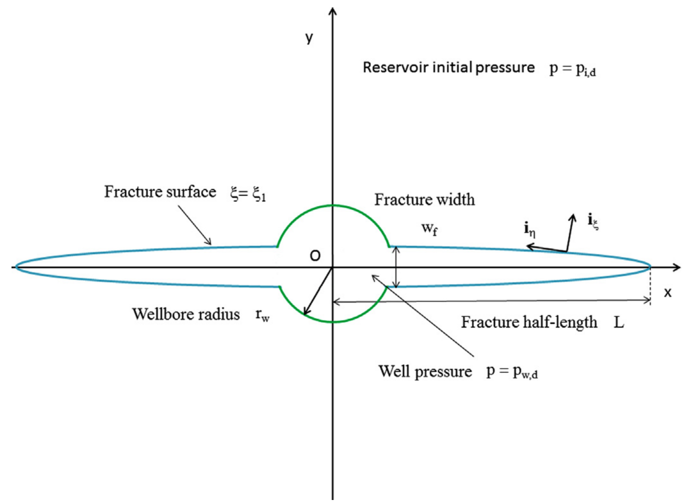

Fig. 1. Top view of a vertical well intersected by an elliptical fracture. The drawing is for illustration purpose only and it does not reflect the actual scales. In practice, the fracture is very thin and long, $L \gg w_f$, $\xi_1 \approx 0$.

Flows in the reservoir and the fracture are governed by mass conservation and the Darcy's law. In the reservoir, we have

$$\frac{\partial(\rho_m\phi_m)}{\partial t} + \mathrm{div}(\rho_m \mathbf{v}_m) = 0, \mathbf{v}_m = -\frac{\kappa_m}{\mu}\nabla p_m, \tag{1}$$

where ρ, μ, $\mathbf{v}_m$, p, ϕ are the fluid density, viscosity, Darcy velocity, pressure and medium porosity. In the traditional petroleum engineering literature, reservoir fluid is considered weakly compressible and it is customary to write

$$\frac{\partial(\rho_m\phi_m)}{\partial t} = c_{tm}\rho_m\frac{\partial p_m}{\partial t}, \tag{2}$$

with the so-called total compressibility c_{tm} given by

$$c_{tm} = \frac{\phi_m}{\rho_m}\frac{\partial\rho_m}{\partial p_m} + \frac{\partial\phi_m}{\partial p_m} = \frac{\phi_m}{K_f} + \frac{\partial\phi_m}{\partial p_m}, \tag{3}$$

where K_f is the fluid bulk modulus. The first term on the right-hand-side of (3) represents the compressibility of the reservoir fluid, and the second term represents the porosity change caused by the depletion of the reservoir pressure. In general, porosity change is related to the deformation of the solid matrix which is determined from the theory of poroelasticity [17–19]. Porosity of the matrix is a function of the effective pressure σ_{eff} which is the difference between the confining pressure $\bar{\sigma}$ and the pore fluid pressure p_m. In general [20]

$$\frac{\partial\phi_m}{\partial p_m} = \frac{d\phi_m}{d\sigma_{eff}}\frac{\partial\sigma_{eff}}{\partial p_m}, \frac{d\phi_m}{d\sigma_{eff}} = \frac{1-\phi_m}{K_{fr}} - \frac{1}{K_s}$$

where K_{fr},K_s are the frame and solid-grain bulk modulus, respectively. The change of the effective stress with the reservoir pressure $\partial\sigma_{eff}/\partial p_m$ has to be obtained from the poroelasticity equation for the effective stress, which for unsteady problems necessarily introduces a two-way coupling between the flow and the matrix deformation [19]. However, this change of the effective stress with the reservoir pressure occurs over the production time scale; and in the conventional theory of petroleum engineering, it is customary to approximate this change as a constant. As a result, the total compressibility is treated as a lumped, constant parameter. This approach not only linearizes the term expressed by Eq. (2) but also greatly simplifies the problem as it decouples the fluid flow problem from the solid deformation problem. While not rigorous, this lumped-parameter approach serves as a good first-order approximation to a complex problem with refinement of the results