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A pragmatic approach to accommodate in-plane anisotropy in forming limit diagrams

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ABSTRACT

The traditionally used forming limit diagram (FLD), a locus of limit strain states under different linear strain paths is usually determined from the blanks along the rolling direction. The effect of in-plane anisotropy on the forming limit diagram is neglected for engineering applications. However, the in-plane anisotropy effect is significant when the FLD is used to estimate forming limit stress diagram (FLSD). The available models to account for the in-plane anisotropy are reviewed and their limitations are discussed. A simpler and more pragmatic approach to account for the in-plane anisotropy in forming limit diagrams is proposed. The new method calculates the change in the plane strain limit using constitutive models of plasticity theory. The orientation specific FLD is then calculated by interpolating between the limit strains along the uniaxial, plane and biaxial strain paths. The proposed methodology is discussed by comparing the experimental FLDs for three different grades of low carbon steels and an aluminum alloy. The calculation of FLSD using the predicted anisotropic FLD, assuming Hill48 yield criterion is illustrated for one of the materials.

1. Introduction

The forming limit diagram (FLD), a plot of limit strains in principal strain space is used to evaluate the formability of metals under linear strain paths. The FLD is experimentally determined through limiting dome height tests where a hemispherical punch is used to deform a sheet until fracture. The strain state near the fracture region is called the forming limit strain. The limiting strains along different strain paths can be obtained by varying the initial blank size. In general, the FLD is determined from blanks whose major axis is along the rolling direction. This FLD is unique for materials exhibiting in-plane isotropy of mechanical properties. The FLD varies with orientation for sheet metals exhibiting in-plane anisotropy. While the orientation effect on limit strains has raised academic curiosity (Rees, 1996), practical usage of orientation specific FLDs have not been practically used to evaluate the formability of blanks with different orientation.

The negligence of anisotropy in FLD could be due to two major reasons, viz. (i) the statistical nature (scatter) of limit strains (Narayanasamy and Sathiya Narayanan, 2006; Strano and Colosimo, 2006) (refer to Fig. 1) and (ii) the linearity of strain paths in FLD. The orientation sensitive limit strains are practically within the experimental scatter, and the influence of in-plane anisotropy cannot be distinguished clearly. With the advancements in strain measurement techniques, such as digital image correlation (Kim et al., 2013), it is expected that the scatter will be reduced and the anisotropic effect may be pronounced in experimental FLDs. The other important reason is that the FLD is valid only when the deformation strain paths are linear or proportional. However, most of the applications (Hariharan et al., 2009; Hariharan and Balaji, 2009) involve non-proportional deformation with changes in strain path during deformation (Graf and Hosford, 1994). Therefore, the FLD loses its physical significance as a tool to evaluate formability.

The forming limit stress diagram (FLSD), on the other hand is proven to be practically insensitive to strain path change over a wider range of strain (Stoughton and Zhu, 2004). With the increased utilization of finite element methods to evaluate formability, the application of FLSD has gained credibility in recent years (Hashemi et al., 2009; Zimniak, 2000). The FLSD is calculated from a strain based forming limit curve (FLC) and plasticity relations. Owing to the nature of stress–strain relations, the forming limits in the stress space are more sensitive than those in the strain space; i.e. a small

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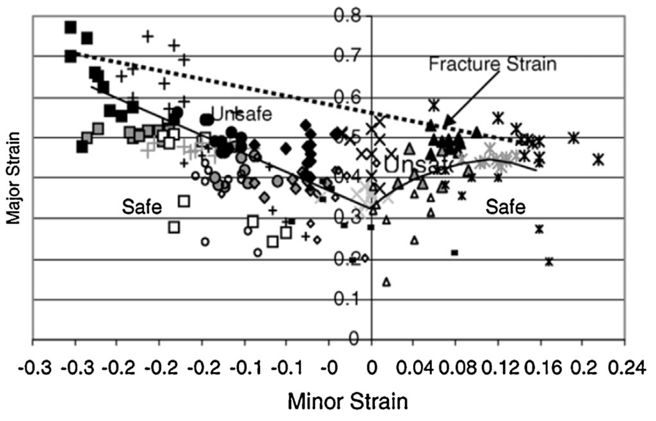

Fig. 1. Experimental FLD of a typical low carbon steel to illustrate the scatter of limit strains (Narayanasamy and Sathiya Narayanan, 2006)

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change in limit strain can introduce a large change in limiting stress values (Stoughton and Yoon, 2012). Therefore, even a minor difference in limit strain value is significant when calculating FLSD from FLC. Under such circumstances, the orientation effect on the limit strains though less in strain space is important for FLSD applications.

Experimental measurement of FLD involves several specimens of different blank widths to estimate the limiting strains under different strain paths. Repetition of the above experiment for each orientation is laborious. This is further complicated by the experimental scatter associated with the experiments. It is therefore advantageous to develop an analytical method to incorporate the effect of in-plane anisotropy so that the FLD along any orientation could be predicted from the traditional FLD along rolling direction. The orientation specific FLD thus predicted could further be used to calculate the FLSD. There have been few attempts in the past to relate the sheet orientation and the FLD along the rolling direction (Rees, 1996; Stoughton and Yoon, 2005). Rees (1996) considered the anisotropy effect in Swift's instability criterion and derived the relation between FLDs along different orientations. The closed form nature of the relation is complex and cannot be extended to advanced non-quadratic yield criteria. Stoughton and Yoon (2005) proposed a simple empirical equation to relate the FLDs along different orientations. Their method needs experimental plane strain limits along the diagonal and transverse directions.

In the present work, a new methodology based on geometric interpolation of the orientation specific limit strains between the uniaxial, plane strain and biaxial strain paths is proposed. The basic methodology is similar to that described by Stoughton and Yoon (2005). However, the variation of the plane strain limit, which is vital for the interpolation, is estimated from the plasticity relations and not from an empirical equation. This provides additional flexibility as the interpolation is material dependent. In addition, the new methodology does not require additional experimental data of plane strain limits along the diagonal and transverse directions. The new methodology is simpler to implement and robust when compared to the previous methods. The orientation specific FLDs derived using the proposed methodology are demonstrated for four materials: three different grades of low carbon steel and an aluminum alloy, Al-6061, reported in the literature (Butuc et al., 2006, 2011; Narayanasamy and Sathiya Narayanan, 2005). The results of the new methodology are compared to those predicted using (Stoughton and Yoon, 2005). In the following section, the previous methods are briefly reviewed, and the proposed methodology is introduced and discussed.

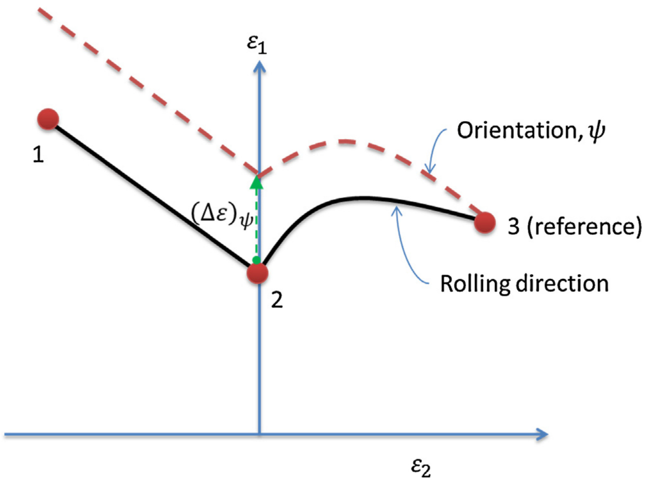

Fig. 2. Geometric interpolation of FLD-schematic. The reference points (1), (2) and (3) refers to uniaxial, plane and equibiaxial strain limits. Equibiaxial strain (3) is fixed as reference.

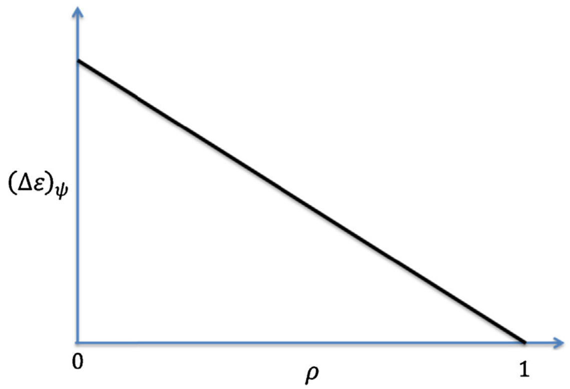

Fig. 3. Linear interpolation of strain increment in right side of FLD – schematic.

2. Orientation specific FLD

The anisotropy effect on FLD has not gained much attention in the literature. Experiments along the transverse direction (Graf and Hosford, 1994) indicated that the FLDs are anisotropic in nature. Using M–K (Marciniak–Kuczynski) analysis, the FLD along the transverse direction was estimated by changing the major reference axis (Aretz, 2007; Barata da Rocha et al., 1985; Kuroda and Tvergaard, 2000). Detailed analytical descriptions of orientation specific limit strains, however, were not developed until Rees (1996).

2.1. Rees (1996) method

Rees modified Swift's instability criterion to account for the effect of the initial blank orientation. The procedure followed by Rees is presented below (the presentation structure and notation are altered from the original paper for generality). Let us consider a plane stress state whose anisotropic axes are at an angle θ to the principal stress axes. The stress state in the material axes can then be related to the principal stress components, σ_1 and σ_2 using tensor transformation as

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{bmatrix} = \begin{bmatrix}\cos^2\theta & \sin^2\theta\\ \sin^2\theta & \cos^2\theta\\ -\sin\theta\cos\theta & \sin\theta\cos\theta\end{bmatrix}\begin{bmatrix}\sigma_1\\ \sigma_2\end{bmatrix} \quad (1)$$