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A new nonlocal bending model for Euler–Bernoulli nanobeams

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ABSTRACT

This paper is concerned with the bending problem of nanobeams starting from a nonlocal thermodynamic approach. A new coupled nonlocal model, depending on two nonlocal parameters, is obtained by using a suitable definition of the free energy. Unlike previous approaches which directly substitute the expression of the nonlocal stress into the classical equilibrium equations, the proposed approach provides a methodology to recover nonlocal models starting from the free energy function. The coupled model can then be specialized to obtain a nanobeam formulation based on the Eringen nonlocal elasticity theory and on the gradient elastic model. The variational formulations are consistently provided and the differential equations with the related boundary conditions are thus derived. Nanocantilevers are solved in a closed-form and numerical results are presented to investigate the influence of the nonlocal parameters.

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1. Introduction

The Euler–Bernoulli beam theory dates back to the 18th century. Such a model is based on the assumption that straight lines normal to the midplane before deformation remain straight and normal to the midplane after deformation. The resulting formulation for solving the deflection of local elastic beams is based on a fourth-order differential equation.

A nonlocal continuum model has been introduced by Eringen (2002) to account for small-scale effects by specifying that the stress at a given point is dependent on the stress in neighbouring points of the body.

An alternative methodology is based on an atomistic approach centred on the molecular dynamics and the molecular mechanics (see e.g. Cao and Chen, 2006; Chen and Cao, 2006).

Starting from the study of Peddieson et al. (2003) which developed a nonlocal Euler–Bernoulli beam model, many contributions on this issue have been proposed following a similar approach, that is the nonlocal model is obtained by replacing the stress appearing in the classical equilibrium equations by its nonlocal counterpart (Reddy, 2007; Wang and Liew, 2007; Aydogdu, 2009; Arash and Wang, 2012).

An hybrid nonlocal beam model is developed in (Zhang et al., 2010) by postulating that the strain energy functional involves both local and nonlocal curvatures. Accordingly such a hybrid model shows nonlocal effects for an Euler–Bernoulli cantilever nanobeam under a transverse point load while the Eringen model is found to be free of small-scale effects for the same problem (Challamel and Wang, 2008). It is worth noting that, on the basis of a simple analogy (Barretta et al., 2014; Barretta and Marotti de Sciarra, 2014), the nonlocality effect on nanorods and nanobeams, formulated according to the Eringen model, can be simulated by prescribing suitable fields of axial and curvature distortions on corresponding local rods and beams. Hence a general procedure is available to establish if nonlocal nanorods and nanobeams are free of small-scale effects.

In the present paper a new coupled nonlocal model is introduced by a suitable definition of the free energy which depends on two small length-scale parameters and on a participation factor which can make the nanobeam flexible or stiffer. Then nonlocal thermodynamics allows us to build up a consistent methodology to derive the related variational formulation and, as a consequence, the differential relations with the associated boundary conditions can be obtained in a straightforward manner.

The proposed coupled model can be specialized to recover the Eringen model (1983, 1987) and the gradient model (Aifantis, 2003; Papargyri-Beskou et al., 2003; Giannakopoulos and Stamoulis, 2007; Akgöz and Civalek, 2012; Barretta and Marotti de Sciarra, 2013).

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An example of a nanocantilever subjected to a uniform load is illustrated and closed-form solutions are provided in order to investigate the influence of the nonlocal parameters. A comparison among the coupled, Eringen and gradient model is thus performed.

2. Kinematics

An Euler–Bernoulli straight nanobeam occupying a domain V is considered. The cross-section of the nanobeam is denoted by Ω, the centroid axis is indicated by x and the bending plane is defined by the Cartesian axes (x, y) originating at the cross-section centroid. The axis orthogonal to the bending plane is denoted by z and the associated second moment of area is $I = \int_\Omega y^2 dA$.

The displacement field $\boldsymbol{s}$ of the nanobeam and the kinematically compatible deformation field $\boldsymbol{D}$ are then given by

$$\boldsymbol{s}(x,y,z) = \begin{bmatrix} -v^{(1)}(x)y \\ v(x) \\ 0 \end{bmatrix}, \quad \boldsymbol{D}(x,y,z) \begin{bmatrix} -v^{(2)}(x)y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1}$$

where v is the transverse displacement along the y-axis and $\chi := v^{(2)} = -\varepsilon/y$ is the nanobeam bending curvature, with ε axial strain. The apex denotes the derivative along the nanobeam axis x.

3. New nonlocal elastic model

In nonlocal elasticity, the first principle of thermodynamic for an isotropic body can be written in a global form and the second one can be expressed in its usual local form (Eringen and Edelen, 1972; Polizzotto, 2003; Marotti de Sciarra, 2009a).

Accordingly the vanishing of the body energy dissipation can be expressed as follows (Marotti de Sciarra, 2009b; Romano et al., 2010; Marotti de Sciarra and Barretta, 2014)

$$\int_V \sigma \dot{\varepsilon}\, dV = \int_V \dot{\beta}\, dV \tag{2}$$

where β is the Helmholtz free energy of the nanobeam and σ is the nonlocal axial stress. The superscript dot denotes differentiation with respect to the time.

A new nonlocal model for nanobeams is proposed in the present paper by considering the following expression of the free energy:

$$\beta(\varepsilon, \varepsilon^{(1)}) = \frac{1}{2}E\varepsilon^2 + \frac{1}{2}c_1^2 E\varepsilon^{(1)2} + \frac{1}{A}\alpha c_2^2 q \chi(\varepsilon) \tag{3}$$

with E Young modulus, c_1, c_2 nonlocal scalar parameters, α participation factor, A cross-section area and q distributed transverse load intensity. The nondimensional scalar factor α can assume any real value, as shown in the sequel, thus providing the weight of the third nonlocal term in Eq. (3). From an engineering point of view, the nanobeam becomes stiffer or not depending on the assumed value of the parameter α.

Remark 3.1. The expression of the free energy (3) leads to a new variational formulation for nonlocal Euler–Bernoulli nanobeams, described by Eq. (5). The corresponding nonlocal model is conceived as a combination between the nonlocal Eringen and gradient models as pointed out in Remark 3.2. The term $c_2^2 q$ in Eq. (3) is peculiar of the model proposed by Eringen (1983). Indeed, as proved by Barretta and Marotti de Sciarra (2014), the elastostatic problem governing a Euler–Bernoulli nonlocal nanobeam is equivalent to the one of a corresponding local nanobeam subjected to a prescribed bending curvature given by $c_2^2 q/EI$.

The time derivative of the free energy is

$$\dot{\beta}(\varepsilon, \varepsilon^{(1)}) = E\varepsilon\dot{\varepsilon} + c_1^2 E\varepsilon^{(1)}\dot{\varepsilon}^{(1)} + \frac{1}{A}\alpha c_2^2 q \partial_\varepsilon \chi(\varepsilon)\dot{\varepsilon} \tag{4}$$

where ∂_ε is the derivative with respect to the axial strain ε.

Table 1
Boundary conditions pertaining to the considered nanobeam model.

Kinematic boundary conditions	Static boundary conditions
v	$-M^{(1)} + \alpha c_2^2 q^{(1)} = -M_0^{(1)} + c_1^2 M_1^{(2)}$
$v^{(1)}$	$M - \alpha c_2^2 q = M_0 - c_1^2 M_1^{(1)}$
$v^{(2)}$	$0 = c_1^2 M_1$

Substituting the time derivative of the free energy given by Eq. (4) into Eq. (2), using the kinematically compatible deformation field $(1)_2$ and noting that $\partial_\varepsilon \chi(\varepsilon)\dot{\varepsilon} = \dot{v}^{(2)}$, we get the following *variational formulation*

$$\int_0^L M\dot{v}^{(2)}\, dx = \int_0^L M_0 \dot{v}^{(2)}\, dx + \alpha c_2^2 \int_0^L q\dot{v}^{(2)}\, dx + c_1^2 \int_0^L M_1 \dot{v}^{(3)}\, dx \tag{5}$$

where the stress resultant moments are given by

$$(M, M_0, M_1) = -\int_\Omega (\sigma, \sigma_0, \sigma_1) y dA = -\int_\Omega (\sigma, E\varepsilon, E\varepsilon^{(1)}) y dA \tag{6}$$

The expression of the free energy (3) leads thus to the new variational formulation (5) for nonlocal Euler–Bernoulli nanobeams and the corresponding nonlocal model can be considered as a combination between the nonlocal Eringen and gradient models as pointed out in Remark 3.2.

This new coupled model can thus be cast in the framework of the so-called hybrid nonlocal theory proposed in Challamel and Wang (2008), Zhang et al. (2010) where a different coupling between the Eringen and gradient models is provided.

It is worth noting that the importance of providing a variational formulation associated with nonlocal models relies also in the fact that it is the starting point to formulate a nonlocal finite element (see e.g., Marotti de Sciarra, 2013, 2014).

Differential and boundary conditions of equilibrium are recovered by integrating by parts the l.h.s. of Eq. (5) and imposing the equality with the external virtual power. In formulae we get $M^{(2)} = q$ in $[0, L]$ and $T = -M^{(1)} = F$ and $M = \mathscr{M}$ at $x = \{0, L\}$, with T shear force and $(F, \mathscr{M})$ transverse force and couple.

Integrating by parts Eq. (5), the following nonlocal differential relation is provided

$$q - \alpha c_2^2 q^{(2)} = M_0^{(2)} - c_1^2 M_1^{(3)} \tag{7}$$

where the corresponding boundary conditions consistently follow from the related variational principle and are reported in Table 1.

The nonlocal elastic equilibrium equation for nanobeams associated with the considered model can then be provided by expressing the differential equations (7) in terms of the transverse displacement v using Eqs. (1) and (6). In fact, noting the equalities

$$(M_0, M_1) = -\int_\Omega (E\varepsilon, E\varepsilon^{(1)}) y\, dA = (EIv^{(2)}, EIv^{(3)}) \tag{8}$$

being $I = \int_\Omega y^2 dA$ the second moment of area about the z-axis, the governing differential equation for the bending of the nonlocal Euler–Bernoulli nanobeam under distributed transverse loads is

$$c_1^2 EIv^{(6)} - EIv^{(4)} = -q + \alpha c_2^2 q^{(2)} \tag{9}$$

where the related boundary conditions can be obtained from Table 1 and are reported in Table 2, with $T = -M^{(1)}$.

Hence the analysis performed above shows that the free energy (3) yields the sixth-order differential equations (9) governing the bending of the Euler–Bernoulli nanobeam. Accordingly six boundary conditions (three for each end of the nanobeam) are required (see Table 2) and the length-scale parameters appear in the differential equation as well as in the boundary conditions.

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