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Deformation patterning in three-dimensional large-strain Cosserat plasticity

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ABSTRACT

In the framework of the rate-independent large-strain Cosserat theory of plasticity explicit analytic solutions are computed in three space dimensions. It is shown that the micro-rotations can be computed by solving stationary Allen–Cahn equations. While the material parameters are within a certain range, this explains the occurrence of patterning leading to a partitioning of the domain into subsets with approximately constant rotations.

1. Introduction

This article focuses on the theoretical investigation of rotation deformation zones predicted by the large-strain rate-independent Cosserat theory of visco-plasticity. The results extend and confirm the two-dimensional findings in Blesgen (2013). Therein, it had been shown that for suitable boundary conditions *deformation patterning* arises. This term refers to the occurrence of Cosserat deformation zones, i.e. the formation of cells in the material with approximately constant micro-rotations as a consequence of deformation. The proposed mechanism may explain the formation of grains and subgrains in solids. Earlier studies of this topic include the articles by Zeghadi et al. (2005), Forest et al. (2000), Vardoulakis and Sulem (1995) and Oda and Iwashita (1999), where the plasticity of polycrystals and the kinetics of the individual grains were investigated.

From its construction, the Cosserat model is a gradient model. In that, in contrast to other established models in elasto-plasticity as Hill (1998), Miehe (1998) and Simo (1988, 1988), it automatically induces a length scale, with the effect that the localisation zones always have a finite width.

This paper is organised in the following way. Section 2 recalls the formalism of the rate-independent large-strain Cosserat theory in the case that plasticity occurs along given slip systems only. In Section 3, analytic solutions to a three-dimensional shear problem are computed, first for a purely plastic case without elastic deformations, secondly for a purely elastic case without plasticity. In both cases, using a parametrisation of the rotation group $SO(3)$ by Euler angles, it is shown that the rotations can be computed by solving an Allen–Cahn system, a model originally derived for studying phase transitions. The article ends with a discussion of the results.

2. The finite-strain Cosserat model of visco-plasticity

Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with Lipschitz boundary serving as the reference configuration of the undeformed material. The total deformation of the solid is controlled by the diffeomorphism $\varphi : \Omega \to \Omega_t$ where $\Omega_t \subset \mathbb{R}^3$ is the deformed solid at time $t \geq 0$. Since $\varphi(\cdot, 0) = \mathrm{Id}$ it holds $\det(D\varphi(t)) > 0$ for all $t \geq 0$.

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List of symbols

$\Omega \subset \mathbb{R}^3$	reference domain, undeformed solid	(x, t)	space and time coordinates
$\sigma_Y > 0$	yield stress (3)	$h>0$ $h>0$	discrete time step
φ	deformation vector of the solid,	$F = D\varphi$	deformation tensor (1)
F_e	elasticity tensor (1)	F_p	plasticity tensor (1)
R_e	rotation tensor (1)	U_e	(right) stretching tensor (1)
Id	identity tensor	K_e	(right) curvature tensor (2)
W_{st}	stretching energy (1)	W_c	curvature energy (1)
λ, μ	Lamé parameters (8)	μ_c	Cosserat couple modulus (8)
ϱ	dislocation energy constant (4)	μ_2	parameter μ scaled by L_c (9)
L_c	internal length scale (9)	$\lVert\cdot\rVert$	Frobenius matrix norm (10)
$\mathrm{tr}(\sigma)$	trace of tensor σ (10)	σ^t	transpose of tensor σ
$\mathrm{sym}(\sigma)$	symmetric part of σ (8)	$\mathrm{skw}(\sigma)$	skew-symmetric part of σ (8)
γ	single-slip parametrisation of F_p (5)	γ^0	values of γ at time t (3),
κ	dislocation density (1)	κ^0	values of κ at time t (3)
I_p	number of single slip systems	$\beta(t)$	shear parameter (7)
α	parametrisation of R_e in 3D (11)	Q_k	matrices of Euler angles (11)
R_D	Dirichlet boundary values of R_e (3)	α_D	Dirichlet boundary values of α (23)
m_k	slip vector of k-th slip system	n_k	slip normal of k-th slip system
c_k, s_k	acronyms for $\cos(\alpha_k)$, $\sin(\alpha_k)$ (12)	J, J_β	double-well potentials (24), (32)

By the Cosserat approach, the deformation tensor $F := D\varphi$ is multiplicatively decomposed into the plastic part F_p and the elastic part F_e. In turn, F_e is split into a rotation component R_e and a stretching component U_e,

$$F = F_e F_p = R_e U_e F_p. \tag{1}$$

It holds $U_e \in \mathrm{GL}(\mathbb{R}^3)$ and $R_e \in \mathrm{SO}(3)$, where GL is the general linear group of invertible matrices, and

$$\mathrm{SO}(d) := \{R \in \mathrm{GL}(\mathbb{R}^d)|\ \det(R) = 1,\ R^t R = \mathrm{Id}\}$$

denotes the special orthogonal group. In general, U_e is not symmetric and positive definite, in particular the decomposition $F_e = R_e U_e$ is *not* the polar decomposition. By

$$K_e := R_e^t D_x R_e = (R_e^t \partial_{x_k} R_e)_{1 \le k \le 3} \tag{2}$$

the third-order (right) curvature tensor is denoted, $\kappa = (\kappa_0, \ldots, \kappa_{I_p})$ designates the vector of stored dislocation densities, $\sigma_Y > 0$ is the yield stress.

Starting point of the analysis is the unconstrained minimisation problem

$$E_\beta(R_e, \gamma) = \int_\Omega \left[W_{st}(R_e^t D\phi F_p(\gamma)^{-1}) + W_c(K_e) + \varrho \left(\sum_{a=1}^{I_p} |\gamma_a - \gamma_a^0| \right)^2 + \sum_{a=1}^{I_p} |\gamma_a - \gamma_a^0| \left(\sigma_Y - 2\varrho \sum_{a=1}^{I_p} \kappa_a^0 \right) \right] \mathrm{d}x \to \min,\ R_e|_{\partial\Omega} = R_D. \tag{3}$$

This problem originates from Eq. (16) in Blesgen (2013) after using the identity (6) on the dislocation densities stated below, plugging in the simple quadratic energy density of stored dislocations

$$V(\kappa) := \varrho \left(\sum_{a=1}^{I_p} \kappa_a \right)^2, \tag{4}$$

and generalising to $I_p \ge 1$ slip systems.

In (3), E_β represents the mechanical energy of a deformed solid. In deriving this functional, it is assumed that plastic deformations occur only along given slip systems, controlled by a set of parameters $\gamma = (\gamma_1, \ldots, \gamma_{I_p})$ according to

$$F_p(\gamma) = \mathrm{Id} + \sum_{a=1}^{I_p} \gamma_a m_a \otimes n_a. \tag{5}$$

In (5), $m_a, n_a \in \mathbb{R}^3$ denote the slip vectors and slip normals with $|m_a| = |n_a| = 1$, $m_a \cdot n_a = 0$ for all slip systems $1 \le a \le I_p$.

For two sets of initial parameters $\kappa^0 = (\kappa_1^0, \ldots, \kappa_{I_p}^0)$, $\gamma^0 = (\gamma_1^0, \ldots, \gamma_{I_p}^0)$ of the previous time step t, the new quantities (R_e, γ) at time $t + h$ are computed as minimisers of E_β. Then,

$$\kappa_a := \kappa_a^0 - |\gamma_a - \gamma_a^0|, \quad 1 \le a \le I_p \tag{6}$$

is set and (κ, γ) serve as initial values of the next time step. This concept of time-discrete minimisation problems goes back to Ortiz and Repetto (1999). It allows to apply variational methods for the investigation of deformation processes.

Starting from a material free of dislocations, $\kappa(\cdot, 0) = 0$, as a consequence of the hardening law (6), $\sum_{a=1}^{I_p} \kappa_a(t+h) \le \sum_{a=1}^{I_p} \kappa(t) \le 0$ for all times t. Therefore, in (3), $-2\varrho \sum_{a=1}^{I_p} \kappa_a^0 \ge 0$ specifies the increase of the yield stress σ_Y due to stored dislocations.

In deriving (3), the deformations are restricted to the shear case

$$D\varphi(t) = \mathrm{Id} + \sum_{a=1}^{I_p} \beta_a(t) m_a \otimes n_a \quad \text{in } \overline{\Omega} \tag{7}$$