Mechanics Research Communications 62 (2014) 57–65

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Reliability evaluation and control for wideband noise-excited viscoelastic systems

S.L. Wang, X.L. Jin*, Y. Wang, Z.L. Huang

Department of Engineering Mechanics, Zhejiang University, Hangzhou 310027, PR China

ARTICLE INFO

Article history:
Received 13 August 2013
Received in revised form 1 August 2014
Accepted 26 August 2014
Available online 6 September 2014

Keywords:
Viscoelastic system
Reliability
Quasi-optimal bounded control
Stochastic averaging
Stochastic dynamic programming principle

ABSTRACT

Reliability of first-passage type for wideband noise-excited viscoelastic systems and the quasi-optimal bounded control strategy for maximizing system reliability are investigated. The viscoelastic term is approximately replaced by equivalent damping and stiffness separately. By using the stochastic averaging method based on the generalized harmonic functions, the averaged Itô stochastic differential equation is obtained for the system amplitude. The associated backward Kolmogorov equation is derived and solved to obtain the system reliability. By applying the dynamic programming principle to the averaged system, the quasi-optimal bounded control is devised by maximizing system reliability. The application of the proposed analytical procedures and the effectiveness of the control strategy are illustrated through one example.

1. Introduction

A great number of materials, such as metals and alloys at high temperature, rubbers, polymers, and composites, exhibit viscoelastic behaviors. These materials are widely used in mechanical engineering, civil engineering, aerospace engineering, electric engineering, bioengineering and other areas. Different from elastic materials, viscoelastic materials not only can store potential energy due to their elasticity, but also dissipate system energy due to their damping mechanisms. Besides displaying the frequency- and temperature-dependent characteristics, viscoelastic materials are nonlinear with respect to their motion amplitudes (Christensen, 1982; Drozdov, 1998). Consequently, it is a challenge to develop constitutive models to characterize viscoelastic behaviors. Over the past decades, many models have been developed and studied, such as Kelvin–Voigt model, fractional derivative model and so on (Roscoe, 1950; Christensen, 1982; Bagley and Torvik, 1983; Drozdov, 1998; Zhang, 2006; Di Paola and Zingales, 2012). Among them, a simple linear viscoelastic model based on the generalized Maxwell model has been widely adopted.

Viscoelastic systems subject to deterministic and random excitations have been widely investigated (Adhikari and Pascual, 2009, 2011; Palmeri et al., 2004; Muscolino et al., 2005). A large amount of researches have been on the stochastic response and stability. Singh and Abdelnaser (1993) analyzed the random vibrations of externally damped viscoelastic Timoshenko beams under general boundary conditions. Muscolino et al. (2005) evaluated time-domain response of linear viscoelastic system under deterministic and random excitations by the Laguerre polynomial approximation method. By replacing a viscoelastic force by a damping force and a stiffness force, Cai et al. (1998) and Cai and Zhu (2011) studied the viscoelastic system under broad-band excitations with the application of the stochastic averaging method. Ling et al. (2011) proposed a similar method and determined the stability based on the largest Lyapunov exponent method. Soize and Poloskov (2012) studied the transient response of linear viscoelastic systems with model uncertainties and nonstationary stochastic excitation by a time-domain formulation. Ariaratnam (1993) applied the stochastic averaging method to investigate the stability of a viscoelastic system with a linear stiffness and a parametric excitation. Potapov and Koirala (1997) studied the stochastic stability of elastic and viscoelastic systems based on a method of statistical simulation of random processes and on the determination of maximum Lyapunov exponents. Huang and Xie (2008) investigated the stochastic stability by means of the first- and second-order stochastic averaging. Floris (2011) analyzed the stochastic stability of a hinged-hinged viscoelastic column.

* Corresponding author. Tel.: +86 57187952651.
E-mail address: xiaolingjin@zju.edu.cn (X.L. Jin).

http://dx.doi.org/10.1016/j.mechrescom.2014.08.010
0093-6413/

Besides the response prediction and stability analysis, reliability of viscoelastic systems is also an important and meaningful topic. It is known that the reliability is dependent on the failure modes. Among different failure modes, the first-passage failure is a typical one. It says that a system may be damaged or disabled once its state leaves the safety domain for the first time. Amaniampong and Ariaratnam (1998) studied the first-passage problem of a linear viscoelastic system subject to stationary Gaussian process, and found that the presence of viscoelasticity increases the system reliability. Guo et al. (2002) presented a framework for performing seismic reliability analysis of hysteretic structure-viscoelastic damper systems with and without parameter uncertainties, and concluded that the failure probabilities of the structures significantly decreased after the installation of the viscoelastic dampers of suitable parameters. Another possible way to reduce the system response and enhance the system reliability is to add an active control device, which is one of the topics of this investigation. To authors' knowledge, the design of the optimal control to maximize the reliability of viscoelastic system whose motion is described by a stochastic integro-differential equation has not been researched.

In this paper, the method proposed by Ling et al. (2011) is applied to investigate the reliability of a viscoelastic system with a strongly nonlinear stiffness and weakly nonlinear damping, subject to wideband noise excitations, as well as the quasi-optimal bounded control for maximizing system reliability. The stochastic averaging method based on the generalized harmonic functions is applied to simplify the system, and the theory of Markov processes is employed to analyze the reliability of first-passage type. Furthermore, by applying the dynamic programming principle, the quasi-optimal bounded control is derived by maximizing the system reliability. An example is given to illustrate the applicability of the proposed procedures and the efficacy of the control strategy.

2. Stochastic averaging of the viscoelastic system

Viscoelastic beam under random excitations is a practical research topic in structural engineering. The transverse vibration of a viscoelastic beam is governed by a stochastic partial differential equation. Through Galerkin procedure, the fundamental mode of the vibration is governed by the following single-degree-of-freedom equation,

$$\ddot{X} + \varepsilon c(X, \dot{X})\dot{X} + g(X) + \varepsilon KZ = \varepsilon^{1/2} f_k(X, \dot{X})\xi_k(t) \tag{1}$$

where X and $\dot{X}$ are the generalized displacement and velocity, respectively; $c(X, \dot{X})$, $g(X)$ and Z are related to the damping force, stiffness force and the viscoelastic force, respectively; ε is a small positive parameter; K is a constant proportional to the fundamental natural frequency of the linear counterpart; $\varepsilon^{1/2} f_k(X, \dot{X})\xi_k(t)$ ($k = 1, 2, \ldots, m$) represent weak external and/or parametric wideband noise excitations; and the Einstein notation is adopted. In Eq. (1), the viscoelastic term Z is described by the generalized Maxwell model (Roscoe, 1950; Christensen, 1982; Drozdov, 1998; Zhang, 2006)

$$Z = \int_0^t h(t-\tau)X(\tau)d\tau \tag{2}$$

In the model, function $h(t)$ is called the relaxation function assumed to be

$$h(t) = \dot{G}(t), \quad G(t) = \sum_{i=1}^{m} E_i e^{-t/\lambda_i} \tag{3}$$

where m is the number of Maxwell units in a parallel chain, E_i is the general elastic modulus, and λ_i is the relaxation time.

To proceed further from Eqs. (1)–(3), we use the result developed in Ling et al. (2011), in which the viscoelastic force can be approximately replaced by a damping force and a stiffness force. Then, the viscoelastic system (1) is approximately equivalent to the following system without viscoelasticity,

$$\ddot{X} + \varepsilon c_1(X, \dot{X})\dot{X} + g_1(X) = \varepsilon^{1/2} f_k(X, \dot{X})\xi_k(t) \tag{4}$$

where

$$\begin{aligned} c_1(X, \dot{X}) &= c(X, \dot{X}) + K\sum_{i=1}^{m} \frac{E_i\lambda_i}{1+(\lambda_i\bar{\omega})^2} \\ g_1(X) &= g(X) - \varepsilon K \sum_{i=1}^{m} \frac{E_i X}{1+(\lambda_i\bar{\omega})^2} \end{aligned} \tag{5}$$

in which $\bar{\omega}$ is the average frequency depending on the vibration amplitude of the corresponding Hamiltonian system. The expression of $\bar{\omega}$ depends on the specific system, see Ling et al. (2011) and the example herein. The additional stiffness and damping terms in Eq. (5), depending on the viscoelastic parameters E_i and λ_i, describe effects of the viscoelastic behavior.

Next, the stochastic averaging method is applied to study system (4). Introducing the following transformation (Zhu et al., 2001),

$$X(t) = A\cos\Theta(t) + B, \quad \dot{X}(t) = -A\nu(A, \Theta)\sin\Theta(t), \quad \Theta(t) = \Phi(t) + \Gamma(t) \tag{6}$$

where the instantaneous frequency is

$$\nu(A, \Theta) = \frac{d\Phi}{dt} = \sqrt{\frac{2[U(A+B) - U(A\cos\Theta + B)]}{A^2\sin^2\Theta}} = b_0(A) + \sum_{i=1}^{\infty} b_i(A)\cos i\Theta \tag{7}$$