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Vibration properties of piezoelectric square lattice structures

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ABSTRACT

This work is devoted to study the dynamic problems of periodic piezoelectric structures by using the spectral element method (SEM). The dynamic stiffness matrix of the piezoelectric square lattice is formulated by this method. Highly accurate frequency-domain solutions of the lattices with and without piezoelectric beams are obtained. Band gap properties of the piezoelectric square lattices are investigated and the influences of the different contents of piezoceramic layers are analyzed. The behaviors of the waveguide due to material defects are demonstrated. Lattice structures with different central and peripheral parts are designed and their unique dynamic characteristics are also analyzed.

1. Introduction

Elastic wave propagation in periodic media leads to special dynamic properties, such as vibration band-gaps. Due to the band-gap characteristics, periodic structures are considered as functional materials or structures and have received much experimental and theoretical attention in recent years (Golub et al., 2012; El-Naggar et al., 2012; He et al., 2013; Tee et al., 2010). For some frequency ranges, the vibration propagation in such periodic structures is forbidden. These frequency ranges are called stop-bands or band-gaps (Leamy, 2012; Gazalet et al., 2013). The elastic waves in some other frequency ranges can propagate in the structure and the frequency ranges are called pass-bands. Periodic structures have potential applications such as in frequency filtering, noise suppression, vibration isolation, and design of novel transducer and acoustic devices.

In recent years, with the increasing attention on the investigation of elastic wave propagation in periodic structures, several methods have been developed to analyze the band-gap characteristics. Wang et al. (2010) analyzed the stop-band characteristics of elastic waves in piezoelectric phononic crystals by the plane wave expansion method. Li et al. (2013) calculated the elastic wave band-gaps of two dimensional (2D) phononic crystals composed of square or triangular lattices of solid cylinders in a solid matrix by the boundary element method. Besides, lump mass method (Wang et al., 2005), multiple scattering theory (Sainidou et al., 2008), finite difference time-domain method (Sun and Wu, 2007) and finite element method (FEM) (Liu and Gao, 2007) were also applied to study the periodic structures. Recently, the spectral element method (SEM) was used to study the band-gap properties due to its unique advantages (Wu et al., 2013a,b).

The SEM is based on the Fourier-transform analysis. For a geometrically and materially uniform structure, it can be considered to be only one spectral element (Doyle, 1997; Lee, 2009). Thus, the element number can be reduced largely. Moreover, the SEM provides exact solutions because the element interpolation functions are based on the eigenfunctions of the structural equation of motion. Instead of the simple polynomials in the finite element method, the spectral element applies the frequency-dependent interpolation functions. Due to these advantages, there are growing interests in the SEM applied to various structures (Wu et al., 2013a,b; Żak, 2009; Banerjee et al., 2008).

Piezoelectric beams are often utilized as energy harvesters or wafer piezoelectric transducers. Piezoelectric materials have also received considerable attention due to their wide range applications in the active control of engineering materials and structures. Erturk and Inman (2011) adopted the lumped parameter model, the Rayleigh–Ritz method and the distributed parameter model to analyze energy harvesting by means of the piezoelectric beam. Wang (2012) studied the bimorph piezoelectric beam harvester based on the Timoshenko and Euler–Bernoulli beam theories. Lee et al. (2013) studied a laminated composite beam with a piezoelectric

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layer and discussed the effect of piezoelectric layer on the dispersion relation of both symmetrically and asymmetrically laminated composite beam. They also revealed that the piezoelectric layer reduced the group velocities of all the wave modes. Jang et al. (2014) investigated a smart beam with surface-bonded piezoelectric transducers and analyzed the effect of structural damping on the propagation of shear waves. As far as we know, studies on vibration band-gap properties of lattices with piezoelectric beams have not been reported yet in literature.

In this paper, the SEM is further developed and applied to investigate the dynamic properties of piezoelectric square lattice structures. Based on the Timoshenko beam theory, the dynamic stiffness matrices of the beam element with and without piezoceramic layers are deduced. The spectral equation of the whole lattice structure is established. After the validation of the proposed SEM, the band-gap and defect-state characteristics are presented and analyzed. Some interesting findings are reported and discussed.

2. Problem description

In this section, the 2D square lattice in the global coordinate system (x^g–y^g) as shown in Fig. 1(a) is considered. It contains 15×15 repeating unit cells and the corresponding unit cell is displayed in Fig. 1(b). The unit cell is made of two materials. The black part is material M_1 and the gray part is material M_2. The lengths of the two materials are l_1 and l_2, respectively.

The in-plane (x_g–y_g plane) vibrations are studied. There are altogether three degrees of freedom (DOFs) per node. The unit cell is considered as four beams which contain bending and tension components. For the case of small elastic deflection, the uncoupled superposition of bending and tension vibrations can be carried out.

The spectral stiffness matrices of an elastic beam element and a piezoelectric beam element are deduced by the SEM in Sections 3.1 and 3.2. For the conventional pure elastic beam, it is homogeneous, isotropic, elastic and has a uniform thickness. For the piezoceramic beam, its deformation is assumed to be small and it exhibits linear piezoelectric material behaviors.

In the SEM, the dynamic stiffness matrix of each beam is firstly deduced in its local coordinate system (x–y). Then the dynamic stiffness matrix can be transformed from the local coordinate system into the global one by the transformation matrix. Finally, the dynamic stiffness matrix of the whole structural system can be assembled. In this method, treating the elements separately makes it possible to analyze the structure consisting of an arbitrary number of elements, and the transformation from local to global coordinates allows the beams to be connected at any orientations.

3. Spectral element method

3.1. Elastic beam element

The free vibration of the Timoshenko beam is described by

$$\rho \frac{\partial^2 u(x,t)}{\partial t^2} - E \frac{\partial^2 u(x,t)}{\partial x^2} = 0, \tag{1}$$

$$\kappa GA \left[\frac{\partial^2 v(x,t)}{\partial x^2} - \frac{\partial \theta(x,t)}{\partial x} \right] - \rho A \frac{\partial^2 v(x,t)}{\partial t^2} = 0, \tag{2}$$

$$EI \frac{\partial^2 \theta(x,t)}{\partial x^2} + \kappa GA \left[\frac{\partial v(x,t)}{\partial x} - \theta(x,t) \right] - \rho I \frac{\partial^2 \theta(x,t)}{\partial t^2} = 0, \tag{3}$$

where $u(x, t)$ and $v(x, t)$ is the longitudinal and transverse displacements, $\theta(x, t)$ is the rotation, ρ is the mass density, E is the Young's modulus, $G = E/[2(1 + \upsilon)]$ is the shear modulus with υ being the Poisson's ratio, I is the area moment of inertia about the bending axis, and κ is the shear correction factor depending on the shape of the cross section (Timoshenko and Gere, 1972).

Table 1
Correspondence of matrix elements.

$\boldsymbol{S}$	$\boldsymbol{S}_v$	$\boldsymbol{S}_u$
(1,1) (1,4)		(1,1) (1,2)
(2,2) (2,3)	(1,1) (1,2)	
(2,5) (2,6)	(1,3) (1,4)	
(3,2) (3,3)	(2,1) (2,2)	
(3,5) (3,6)	(2,3) (2,4)	
(4,1) (4,4)		(2,1) (2,2)
(5,2) (5,3)	(3,1) (3,2)	
(5,5) (5,6)	(3,3) (3,4)	
(6,2) (6,3)	(4,1) (4,2)	
(6,5) (6,6)	(4,3) (4,4)	

The general solutions to Eqs. (1)–(3) can be given by the spectral representation (Doyle, 1997)

$$u(x,t) = U(x,\omega)e^{i\omega t}, \tag{4}$$

$$v(x,t) = V(x,\omega)e^{i\omega t}, \tag{5}$$

$$\theta(x,t) = \Theta(x,\omega)e^{i\omega t}, \tag{6}$$

where $U(x, \omega)$, $V(x, \omega)$ and $\Theta(x, \omega)$ are the spectral displacements of u, v and θ.

Substituting Eqs. (4)–(6) into Eqs. (1)–(3), one can obtain the governing differential equations in the frequency-domain. Base on these equations and definitions of the nodal displacements and forces, the relations between the spectral nodal displacements and forces can be written as (Doyle, 1997; Lee, 2009)

$$\boldsymbol{F} = \boldsymbol{Sd}, \tag{7}$$

where $\boldsymbol{F} = [\,F_x^1 \quad F_y^1 \quad M_y^1 \quad F_x^2 \quad F_y^2 \quad M_y^2\,]^T$ and $\boldsymbol{d} = [\,U^1 \quad V^1 \quad \Theta^1 \quad U^2 \quad V^2 \quad \Theta^2\,]^T$ are the nodal force and displacement vectors as shown in Fig. 2, and $\boldsymbol{S}$ is the complete dynamic stiffness matrix witch is composed of $\boldsymbol{S}_u$ and $\boldsymbol{S}_v$ (Doyle, 1997; Lee, 2009). The relation between these three dynamic stiffness matrices is shown in Table 1. The terms of $\boldsymbol{S}$ which are not shown in the table are equal to 0.

3.2. Piezoelectric beam element

The piezoelectric beam as shown in Fig. 3 is considered now. The piezoceramic layers are perfectly bonded on the base beam. The length and width of the piezoelectric beam are L and b, and the thicknesses of the base beam and piezoceramic layers are h_b and h_p, respectively. The top and bottom piezoceramic layers are poled in the opposite thickness directions (Erturk and Inman, 2011; Wang, 2012). The series connection of the electrical circuit is shown in Fig. 3(c). As shown in Section 3.1, the dynamic stiffness matrix of the tension component is uncoupled with that of the bending component. In this subsection, pure bending motion is the main objective to be discussed.

The piezoelectric beam element is considered to be a Timoshenko beam. So the constitutive equations of the piezoelectric layers can be given in matrix form as

$$\begin{Bmatrix} \varepsilon_p \\ \gamma_p \\ D_3 \end{Bmatrix} = \begin{bmatrix} S_{11}^E & 0 & d_{31} \\ 0 & S_{55}^E & 0 \\ d_{31} & 0 & \varepsilon_{33}^T \end{bmatrix} \begin{Bmatrix} \sigma_p \\ \tau_p \\ E_3 \end{Bmatrix}, \tag{8}$$

where ε_p and γ_p are the normal and shear strains, σ_p and τ_p are the normal and shear stresses, D_3 is the electric displacement, E_3 is the electrical field intensity in the piezoceramic layer across the thickness, S_{11}^E and S_{55}^E are the elastic compliance constants, d_{31} is the piezoelectric constant, and ε_{33}^T is the permittivity constant.