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Calculation of wave structure of the ultrasonic beams in nondestructive testing of brick masonries

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ABSTRACT

A strict mathematical approach is proposed to calculate the wave field of the ultrasonic beam used to test arising defects in the stone or brick masonry structures. The problem is reduced to a certain integral equation with respect to the distribution of the contact stress over the base of the US probe, in frames of the dynamic elasticity theory. There is given a comparison of the so-constructed solution with some approximate theories, including a scalar model. Finally, it is evaluated the possibility to use the US beams of the calculated geometry to detect defects in the masonry structures.

1. Introduction

Many old buildings constructed on the basis of stone or brick masonries require special monitoring. The typical defects in such structures are cracks, voids, inclusions, exfoliations, etc. Frequently, Ground Penetrating Radars are used to detect such defects in the masonry structures (Daniels, 2004). This permits application of the powerful methods of computational tomography, connected with Radon transformation and some other similar ideas (Natterer, 1986). They are founded on electromagnetic or ultrasonic (US) techniques, providing visualization of the internal structure of soil or masonry buildings (Binda et al., 2001; Liseno and Pierri, 2002; Claerbout, 1985).

Very often defects in the masonry structures appear to be strong scatterers. In this case the diffraction should be treated strictly on the basis of Finite Element (FEM) or Boundary Element (BEM) methods (Jami and Polyzakis, 1981; Colton and Kress, 1992). Let us consider a sample masonry block consisting of a number of bricks. Then the defects like cracks may arise between neighbor bricks, so that there are many potential segments for location of such defects.

If such an obstacle is located crossing the US beam, then in the through-transmission method, when passing from the source to the receiving transducer, the defect is well detected. Otherwise, this cannot be detected by a probe. Therefore, the principal goal of the present work is to evaluate the geometry of the US beam, in order to predict whether a certain defect can be detected from a chosen position of the scanning probe.

The calculation of the US beam geometry is a classical problem thoroughly investigated (some fundamentals can be found in Krautkramer (1990)). Usually, this is treated as an appropriate integral calculated over the base of the transducer, by using the hypothesis that either load or vertical velocity is uniformly distributed over the base – in the scalar model as well as in the dynamic elasticity theory (Achenbach, 1973). Obviously, such a hypothesis is valid for relatively high frequencies only when the wave length is considerably shorter than the length of the transducer. In the case of US detection of the brick masonry structures a typical frequency f varies from 20 to 50 KHz. The average longitudinal US wave speed in the brick masonry may be accepted around $c_\ell = 3650$ m/s, and the transverse one – near 40% of the latter, i.e. around $c_t = 1460$ m/s. Then the longitudinal wave length $\lambda_\ell = c_\ell/f$ is somewhere between 7 and 18 cm, and the transverse wave length $\lambda_t = c_t/f$ between 3 and 7 cm. All these quantities are of the same order, not significantly shorter, when compared with the typical size of the US probe's base. Hence, the second goal of the work is to calculate the radiated field in frames of a strict theory, by refusing the hypothesis of uniform contact pressure.

It should be noted that some advanced theoretical models for US nondestructive testing and related diffraction problems have been proposed in recent works (Scalia and Sumbatyan, 1999; Brigante and Sumbatyan, 2010; Sumbatyan et al., 2011; Brigante, 2013).

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2. Mathematical transformations determining the structure of the irradiated US beam

Let us assume that a normal US probe is placed on a free surface of the brick masonry. The base of the probe is assumed to be rigid enough and the thin space between the probe base and the structure to be filled of a liquid lubricating layer. In this case there is no tangential stress in the contact zone.

Let us start from classical equations of dynamic elasticity. If one considers a harmonic oscillations of the transducer with the time-dependent factor $e^{-i\omega t}$, then in the 2nd case the displacement vector written in the rectangular Cartesian coordinate system xy is $\bar{u} = \{u_x(x, y),\ u_y(x, y),\ 0\}$, and the only non-trivial components of the stress tensor are: $\sigma_{xx}(x, y), \sigma_{yy}(x, y), \tau_{xy}(x, y)$. Further, the equations of motion are described by the following system of partial differential equations (Achenbach, 1973)

$$\begin{cases} \dfrac{\partial^2 u_x}{\partial x^2} + c^2 \dfrac{\partial^2 u_x}{\partial y^2} + (1 - c^2)\dfrac{\partial^2 u_y}{\partial x \partial y} + k_\ell^2 u_x = 0, \\ \dfrac{\partial^2 u_y}{\partial y^2} + c^2 \dfrac{\partial^2 u_y}{\partial x^2} + (1 - c^2)\dfrac{\partial^2 u_x}{\partial x \partial y} + k_\ell^2 u_y = 0, \end{cases} \tag{2.1}$$

where

$$k_\ell = \frac{\omega}{c_\ell}, \quad k_t = \frac{\omega}{c_t}, \quad c^2 = \frac{c_t^2}{c_\ell^2} = \frac{k_\ell^2}{k_t^2} < 1, \tag{2.2}$$

k_ℓ and k_t are longitudinal and transverse wave numbers, respectively.

The components of the stress tensor are expressed through functions u_x and u_y as follows

$$l\frac{\sigma_{xx}}{\rho c_\ell^2} = \frac{\partial u_x}{\partial x} + (1 - 2c^2)\frac{\partial u_y}{\partial y}, \quad \frac{\sigma_{yy}}{\rho c_\ell^2} = (1 - 2c^2)\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}, \quad \frac{\tau_{xy}}{\rho c_t^2} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}. \tag{2.3}$$

where ρ is the mass density of the material.

If the probe is rigid enough then one may model its oscillations as a vertical vibration of an absolutely rigid punch with a frictionless contact. Then the boundary conditions are

$$y = 0: \qquad \tau_{xy} = 0, |x| < \infty; \quad \sigma_{yy} = 0, |x| > a; \quad u_y = u_0, |x| < a; \tag{2.4}$$

where u_0 is the amplitude of vertical vibration, and $(-a, a)$ is the contact zone.

In order to solve system (2.1) with boundary conditions (2.4), let us apply the Fourier transform along x-axis. System (2.1) in Fourier images is

$$\begin{cases} c^2 U_x'' - s^2 U_x + (1 - c^2)(-is)U_y' + k_\ell^2 U_x = 0, \\ (1 - c^2)(-is)U_x' + U_y'' - c^2 s^2 U_y + k_\ell^2 U_y = 0, \end{cases} \tag{2.5}$$

where capital letters designate Fourier images of respective originals and ordinary derivatives are applied with respect to variable y.

Let $\sigma_{yy}(x, 0) = \sigma(x)$. Note that $\sigma(x) = 0$, $|x| > a$, and the unknown quantity $\sigma(x)$, $|x| \le a$ physically means the contact pressure. Then the introduced new function, together with boundary condition (2.4), leads to the following relations ("tilde" means the sign of equivalence):

$$y = 0: \quad \tau_{xy} = 0, \sim \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = 0;$$

$$\sigma_{yy} = \sigma(x), \sim (1 - 2c^2)\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = \frac{\sigma(x)}{\rho c_\ell^2}, \tag{2.6}$$

which in the space of Fourier transforms are:

$$U'_x - isU_y = 0, \quad (1 - 2c^2)(-is)U_x + U'_y = \frac{\Sigma(s)}{\rho c_\ell^2}. \tag{2.7}$$

The general solution to system (2.5) can be represented in the following form

$$\begin{bmatrix} U_x \\ U_y \end{bmatrix} = C_1 \begin{bmatrix} is/\gamma \\ 1 \end{bmatrix} e^{-\gamma y} + C_2 \begin{bmatrix} -q/(is) \\ 1 \end{bmatrix} e^{-qy},$$

$$\gamma = \sqrt{s^2 - k_\ell^2}, \quad q = \sqrt{s^2 - k_t^2}, \tag{2.8}$$

where C_1, C_2 are some unknown constants.

The substitution of functions U_x, U_y (2.8) into boundary conditions (2.7) defines the value of coefficients $C_1(s), C_2(s)$:

$$C_1 = -\frac{\gamma(s)\Sigma(s)}{\rho c_t^2 \Delta(s)}(2s^2 - k_t^2), \quad C_2 = \frac{2s^2\gamma(s)\Sigma(s)}{\rho c_t^2 \Delta(s)},$$

$$\Delta(s) = (2s^2 - k_t^2)^2 - 4s^2\gamma(s)q(s) \tag{2.9}$$

where $\Delta(s)$ is the well known Rayleigh function (Achenbach, 1973). It is evident from Eqs. (2.12) and (2.14) that

$$U_y(s, 0) = C_1(s) + C_2(s) = \frac{k_t^2 \gamma(s)\Sigma(s)}{\rho c_t^2 \Delta(s)}, \tag{2.10}$$

whose Fourier inversion, with the use of the convolution theorem and the last boundary condition in (2.4), leads to the basic integral equation for contact pressure $\sigma(x)$:

$$\int_{-a}^{a} \sigma(\xi)K(x - \xi)d\xi = \rho c_t^2 u_0, \quad |x| < a, \tag{2.11a}$$

$$K(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} L(s)e^{-ixs}\,ds = \frac{1}{\pi}\int_0^{\infty} L(s)\cos(xs)ds, \quad L(s) = \frac{k_t^2\gamma(s)}{\Delta(s)}. \tag{2.11b}$$

So long as contact pressure, function $\sigma(x)$, is defined from Eq. (2.11), all physical quantities can be found by using Eqs. (2.8), (2.9), and (2.3). In particular, one can easily calculate the amplitude of vertical oscillations of the receiving transducer placed on the opposite side of the specimen and the amplitude of its horizontal oscillations when it is placed on the lateral side of the specimen. Note that the normal transducer operates with longitudinal waves. Then in all formulas only terms with exponential function $e^{-\gamma y}$ should only be kept, neglecting the terms with e^{-qy}.

The quantities in concern are given by the following expressions:

$$\rho c_t^2 u_{x,y}(x, y) = \int_{-a}^{a} \sigma(\xi)K_{x,y}(x - \xi)d\xi, \quad K_x(x) = \frac{1}{\pi}\int_0^{\infty} L_x(s)\sin(xs)e^{-\gamma(s)y}\,ds,$$
$$K_y(x) = \frac{1}{\pi}\int_0^{\infty} L_y(s)\cos(xs)e^{-\gamma(s)y}ds, \; L_x(s) = \frac{s(k_t^2 - 2s^2)}{\Delta(s)}, \quad L_y(s) = \frac{\gamma(s)(k_t^2 - 2s^2)}{\Delta(s)}. \tag{2.12}$$

where representations for kernels K_x and K_y are valid inside the specimen, for $y > 0$.