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Electromechanical instability in layered materials

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ABSTRACT

This paper deals with instability of a semi-infinite strip of polarizable layered material which is subjected to both a boundary displacement and an externally applied electrostatic potential in a plane deformation setting. Since the material is polarizable, it contributes (here in a linear fashion) to the applied electrostatic field. The nonlinear equilibrium problem is solved through a perturbative scheme and the Euler–Lagrange equations are presented. Closed-form solutions are found for some special situations and they are checked against some established results. It is shown that the general condition which lends the instability threshold is obtained enforcing that a third degree polynomial admits a double negative real solution. This amounts to seeking the roots of the discriminant of the polynomial and to checking two conditions. The negative double root yields the perturbation frequency. In the general case, a numerical solution is called upon and an instability curve, in terms of electrostatic potential vs. boundary displacement at threshold, is found. At reaching such curve, the material suddenly superposes to a homogeneously stretched configuration a periodic undulation in both the displacement and the electrostatic fields. A parametric analysis is put forward and an interesting non-monotonic behavior is found. The frequency as well as the amplitude of both the mechanical and the electrostatic undulations are found and discussed.

1. Introduction

Layered materials embody a wide class of different actual materials ranging from thin magnetic films to lipid vesicles, plastic sheets and liquid crystals. All these materials share the common feature that, in a suitable range of behavior, their mechanics may be effectively described through a dilatative strain energy across and a bending energy along the layers. In particular, a large amount of literature is devoted to studying smectics-A liquid crystals as layered materials endowed with a unit vector microstructure which is related to their optical property (Prost and de Gennes, 1993). It is found that a liquid crystal sample, when subjected to an external perturbation, reaches a threshold deformation beyond which it abruptly changes its optical properties (Meyer and Clark, 1973). This phenomenon, which possesses experimental relevance, is called the Helfrich–Hurault effect (Prost and de Gennes, 1993, Section 7.1.6). The source of perturbation which triggers instability may be either mechanical, for instance in the form of a boundary displacement (Ribotta and Durand, 1977; Meyer and Clark, 1973), or electrostatic, through a potential difference between the conducting plates of a capacitor (Helfrich, 1970; Singer, 1993; Napoli and Bevilacqua, 2005), or magnetic, through an externally applied magnetic field (Hurault, 1973; Stewart, 1998), or even thermo-optical, by means of light absorption (Prost and de Gennes, 1993, Section 7.1.8).

So far, the treatment of instability in layered materials is largely confined to the linear theory of deformation and it parallels the pioneering work of Prost and de Gennes (1993). In Mahadevan and Cerda (2003), a much wider perspective is adopted and, moving from thin elastic polyethylene sheets, a general theory of wrinkling is put forward by means of the mechanics of finite deformations in

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layered materials. As pointed out in Brochard-Wyart and de Gennes (2003), such behavior is met in many every day situations and materials, ranging from an old apple skin to human skin and, possibly, geological formations. Following this viewpoint, in Napoli and Nobili (2009) the classical problem of strain induced instability in layered materials is investigated under the theory of finite deformation. It is found that the classical results for the threshold boundary displacement and the undulation amplitude are limiting values of more general expressions, valid inasmuch as the ratio of the material constants (coherence length) is small compared to the sample thickness.

In this paper, instability of a semi-infinite cell of polarizable layered material in a plane deformation layout is studied. The material is confined between the conducting plates of a capacitor and instability arises under the combined effect of both mechanical boundary displacement and electrostatic potential. Finite deformations are considered, such that the resulting equilibrium problem is nonlinear and the superposition principle no longer holds. Nonetheless, as in Mahadevan and Cerda (2003) and Napoli and Nobili (2009), a perturbative approach is pursued so that the equilibrium configuration is thought of as being the superposition of a small mechanical perturbation onto a finitely and homogeneously stretched configuration. Likewise, since the medium is polarizable, the total electrostatic field is given by a linear bias generated by the conducting plates (whose distance depends on the boundary displacement) plus a small perturbation field. Instability occurs when either of the perturbations exists in the form of a periodic function. Results may be useful in designing electro-optic devices employing soft layered materials or in measuring the model's constitutive parameters by inverse analysis (Bahadur, 1992). It is remarked that, as required by the general theory of wrinkling (Mahadevan and Cerda, 2003), kinematical nonlinearity is accounted for. In this respect, the boundary displacement is finite and generally not negligible with respect to the initial thickness, so that, as pointed out in Napoli and Nobili (2009), it affects the successive application of the perturbations. It is further remarked that other forms of instability are equally possible other than that considered herein. Such is the case of, for instance, anisotropic or fiber reinforced materials (Prikazchikov et al., 2008), kink-band instability (Peletier et al., 2004; Wadee and Edmunds, 2005), wherein loading acts along, other than across, the layers, and the case of chevron (or zigzag) instability, which applies to layered materials as well, yet beyond the Helfrich–Hurault threshold (Singer, 1993).

The paper is organized as follows. Section 2 introduces the kinematics of layered materials. Section 3 leads to the free energy expression for polarizable layered materials. The perturbative approach is introduced in Section 4, where the Euler–Lagrange equations for the linearized problem are obtained. Their specialized forms in the cases of no electrostatic potential, no mechanical displacement and isotropic polarization allow closed-form solutions, which may be checked against the established results. Section 5 brings the general solution to the problem through a cubic equation which lends instability inasmuch as it possesses a double and a single real root. The common frequency of the perturbations is given by the location of the double root. The amplitude of the perturbations is obtained at Section 6. A numerical solution is given in Section 7 along with a parametric analysis. Plots of the instability curve, of the frequency relative variation with respect to the classical result and of the dimensionless amplitudes for both the mechanical and the electrostatic perturbations are given. In particular, the amplitudes depend whether the controlling parameter of the problem is the mechanical displacement or the electrostatic potential. Finally, conclusions are drawn in Section 8.

2. Kinematics

A layered material may be regarded as a stack of interacting sheets, each endowed with a rigidity in its plane much higher than the stiffness across. As a result, the kinematics of layered materials may be described through the deformation of iso-surfaces σ_k whose description is in the form (Weinan, 1997; Capriz, 1997; Napoli, 2006)

$$\omega(\boldsymbol{r}, k) = 0. \tag{1}$$

Accordingly, a point lies on the kth layer if its position vector $\boldsymbol{r}$ fulfills Eq. (1). In the natural state, layers are plane and equispaced. In a continuum mechanics approach, the cardinality of k is so large that it is actually replaced by a dense range of values. Furthermore, the deformation is introduced as the one-to-one orientation preserving mapping $\boldsymbol{\chi}$ such that

$$\boldsymbol{r} = \boldsymbol{\chi}(\boldsymbol{r}_0), \tag{2}$$

where $\boldsymbol{r}$ is the position vector, in the actual configuration, of a material particle whose position is $\boldsymbol{r}_0$ in the natural state. The deformation gradient is thus defined

$$\mathbf{F} = \mathrm{Grad}\,\boldsymbol{\chi}, \tag{3}$$

and $j = \det \mathbf{F} > 0$. Layered materials are endowed with a vectorial microstructure given by the unit vector n normal to the layers. In this form, microstructure is entrained by the local motion and it is often referred to as being latent, for its behavior is already described by the local kinematics. Recalling the property of the gradient of a iso-surface, it is

$$\mathsf{n} = \frac{\mathrm{grad}\,\omega}{\|\mathrm{grad}\,\omega\|}, \tag{4}$$

where 'grad' is the spatial gradient and should not be confused with the material gradient 'Grad'. Here, microstructure serves as a mean of specifying the polarization direction of the molecules making up the layers. Polarization along n is referred to as parallel, polarization across as orthogonal.

A plane deformation framework is considered and the deformation plane is spanned by the orthogonal system of unit vectors $\{\mathsf{i}, \mathsf{k}\}$, directed along the x- and z-axis, respectively. The material under consideration, in its natural state, occupies the plane region comprised between the bounds $z = 0$ and $z = d$, i.e.

$$D_0 = \{(x, z) \in \mathbb{R}^2 : 0 \leqslant z \leqslant d\}. \tag{5}$$

and the homogeneous field of unit vectors normal to the layers is taken along the z-axis, i.e. $\mathsf{n}_0 \equiv \mathsf{k}$. Then, Eq. (4) may be rewritten in terms of deformation gradient