Mechanics Research Communications 57 (2014) 34–39

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

An alternative reduced order model for electrically actuated micro-beams under mechanical shock

Amir R. Askari, Masoud Tahani*

Department of Mechanical Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

ARTICLE INFO

Article history:
Received 26 June 2013
Received in revised form 9 November 2013
Accepted 6 February 2014
Available online 15 February 2014

Keywords:
MEMS
Mechanical shock
Electrostatic actuation
Reduced order model

ABSTRACT

This study focuses on the effect of mechanical shock on dynamic pull-in instability of eclectically actuated micro-beams through an alternative reduced order model (ROM). The model's predictions for dynamic pull-in voltages are compared with available finite element (FE) results and six modes Galerkin approximations in the literature. It is shown that present results for high shock accelerations agree with FE predictions better than those obtained using six modes approximations. Furthermore, the present model can remove the limitation of previous methods in capturing dynamic pull-in instability for cases under enormous shock accelerations.

1. Introduction

Dynamic analysis of micro-electro-mechanical systems (MEMS) is a very desirable research topic nowadays. These systems have applications in many engineering fields such as communications, automotive and robotics (Senturia, 2001). Electrically actuated micro-beams can be considered as a building block of these systems (Batra et al., 2007). One of the most essential issues in MEMS design is their reliability under electrical loads (Younis, 2011). Dynamic pull-in instability can be considered as an important source of failure in electrically actuated micro-systems (Younis, 2011). This instability is occurred when the input voltage exceeds a critical value called dynamic pull-in voltage. In this manner the elastic micro-beam suddenly collapses toward the substrate underneath it. In spite of the fact that electrically actuated micro-systems are usually designed far from this instability, dynamic pull-in phenomenon is desired in some cases such as capacitive micro-switches (Rebeiz, 2003).

Mechanical shock can induce highly dynamic loads on structures causing several types of fracture problems. In MEMS, shock loads can cause micro-structures to hit the stationary electrodes underneath them and causing some undesirable problems such as stiction (Tas et al., 1996), short circuits (Tanner et al., 2000) and hence failure in the device's function. The majority of micro-structures are fabricated of silicon or polysilicon which are very tough against bending stresses induced from shock acceleration, so failure in MEMS unlike failure in large scale devices does not due to high stresses (Fang et al., 2004). The most important source of failure in MEMS is stiction and electric short circuits; however the incidents between a movable part and other parts or a substrate may lead to failure due to the severe contact stresses.

A shock can be defined as a force applied suddenly over a short period of time relative to natural period of structure (Meirovitch, 2001). A shock load can be characterized by its maximum value, duration and shape. The shock pulse shape in most of cases can be considered as half-sine (Meirovitch, 2001; Srikar and Senturia, 2002). The response of micro-structures to shock loads has been studied by many researchers. Béliveau et al. (1999) characterized experimentally the response of commercial accelerometers due to shock loads and observed some unexpected responses. Brown et al. (2001) investigated commercial accelerometers and a pressure sensor to high-g tests (g refers to the gravitational constant). They reported peculiar modes of failure under severe shock conditions and concluded that improved dynamic modeling and characterization of MEM devices under shock load are needed. Fan and Shaw (2001) simulated the response of a comb-drive accelerometer subjected to severe dynamic shock loads in all directions. They developed a FE model using the software ABAQUS with full non-linear and contact stress capability and remarked that this problem requires a highly non-linear transient dynamic analysis, which is computationally very expensive. Some authors used equivalent lumped spring-mass model to approximate the dynamic response of micro-structures. Their point of view was proper for rough estimation and could not provide an accurate analysis. For example, Li

* Corresponding author. Tel.: +98 511 8806055; fax: +98 511 8763304.
E-mail address: mtahani@um.ac.ir (M. Tahani).

http://dx.doi.org/10.1016/j.mechrescom.2014.02.002
0093-6413/

and Shemansky (2000) studied the motion of MEM accelerometers during the drop tests. They used both of single degree-of-freedom (SDOF) and distributed-parameter model to calculate maximum deflection of cantilever and hinged-hinged beam. Some researchers analyzed micro-structures based on distributed-parameter models. Fang et al. (2004) investigated the response of a micro-cantilever to a half-sine shock pulse using beam model. They utilized the assumed modes method to calculate displacement and bending stresses of the micro-beam. It is noted that most of the authors investigated the effect of shock pulse lonely and they did not account for the interaction between electrostatic excitation and shock pulse acceleration effect. Younis et al. (2006) accounted for the dynamic interaction between these excitations. They used both SDOF and beam models to investigate the response of micro-beam under combined effect of these two excitations. They used the Galerkin-based ROM to solve the governing equation of the beam. Younis et al. (2007) also analyzed the response of mechanical shock on micro-structures incorporating the effect of packaging. They used six modes approximation in the Galerkin-based ROM to simulate the response of micro-structure to the combination of shock acceleration and electrostatic excitation. They verified their model by comparing its results with those prepared utilizing commercial finite element software ANSYS. It was shown that the combination of a shock load and an electrostatic actuation makes the instability threshold much lower than the threshold predicted, considering the effect of shock alone or electrostatic actuation alone (Younis et al., 2006, 2007). It should be noted that neither the FE nor six modes reduced order (RO) models presented by Younis et al. (2006, 2007) could capture dynamic pull-in instability for cases under shock amplitudes higher than 2400g.

Although many researchers have dealt with the mechanical behavior of micro-beams under impact excitations, the research effort devoted to dynamic pull-in analysis of electrically actuated micro-beams under mechanical shock are very limited. It should be noted that both previous dynamic FE and multi-mode RO models which accounts for the interaction of shock and electrical forces are very computationally expensive (Younis et al., 2006, 2007). In addition, these solution procedures could not capture dynamic pull-in instability for cases under enormous shock accelerations (Younis et al., 2006, 2007). Therefore, an alternative solution with lower run time may be required to capture dynamic pull-in instability in every desired loading case such as cases under enormous shock accelerations. The objective of present work is to establish an alternative ROM to remove these limitations of previous FE and RO models.

The present model is non-linear due to the inherent non-linearity of electrostatic excitation and geometric non-linearity of the von Kármán mid-plane stretching. An alternative single mode Galerkin based ROM is used to convert the partial differential equation of motion to an ordinary differential equation in time which is solved numerically using the fourth order Runge–Kutta method. The model's predictions for dynamic pull-in voltage are validated through direct comparison with those presented in the literature. It is found that the present SDOF model can capture dynamic pull-in instability for systems under enormous shock accelerations. Furthermore, our model can predict dynamic pull-in voltage closer to available FE results than previous multi-mode approximations for cases under high amplitude shock accelerations.

2. Theoretical formulation

Consider a clamped–clamped (CC) micro-beam of length L, width b, thickness h, and density ρ under the combined action of electrostatic excitation and shock pulse force (see Fig. 1). The initial distance between the non-actuated micro-beam and the stationary electrode is d. Also, x, y and z are, respectively, the coordinate along

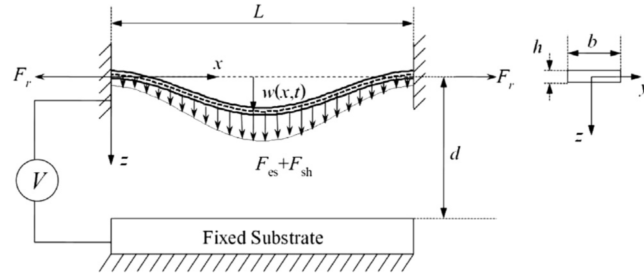

Fig. 1. Schematic of an electrically actuated clamped–clamped micro-beam under the effect of mechanical shock.

the length, width and thickness. Furthermore, w is deflection, t is time, I is the moment of inertia of the cross-sectional area about the y axis and E is the Young's modulus of the micro-beam.

The electrostatic excitation by polarized DC voltage V without the effect of fringing field per unit length of the beam can be expressed as (Batra et al., 2007):

$$F_{es} = \frac{\varepsilon b V^2}{2(d-w)^2} \tag{1}$$

where ε is the dielectric constant of medium. It is noted that the fringing field does not have a sizable effect especially for the case of wide micro-beams (Chao et al., 2008).

The shock force is induced to the micro-structure by an idealized impact acceleration pulse of a half-sine waveform according to JEDEC regulations (JEDEC Solid State Technology Association, 2001, 2003). The shock force is transmitted to the micro-structure through its supports. According to the support excitation scheme (Yeh and Lai, 2006), this base excitation is equivalent to apply the shock acceleration as a distributed force over the micro-structure. A shock pulse force per unit length of the micro-beam F_{sh}, can be defined as $F_{sh} = F_0 g(t)$ where the shock force amplitude F_0 is

$$F_0 = \rho b h a_0 \tag{2}$$

In Eq. (2), a_0 is the amplitude of shock pulse acceleration. The half-sine shock profile can be expressed mathematically as

$$g(t) = \sin\left(\frac{\pi t}{T}\right) U(t) + \sin\left(\frac{\pi}{T}(t-T)\right) U(t-T) \tag{3}$$

where T is the shock duration and $U(t)$ is the unit step function.

By incorporating the von Kármán non-linearity in the expression for the axial strain to account for large deflections and small strains (Landau and Lifshitz, 1986), the equation of motion that governs the transverse deflection $w(x, t)$ of the micro-beam can be written as (Younis et al., 2006)

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = N(w)\frac{\partial^2 w}{\partial x^2} + \frac{\varepsilon b V^2}{2(d-w)^2} + F_0 g(t) \tag{4}$$

where $N(w)$ is the axial force, which is due to an initial axial stress and the elongation of micro-beam called the von Kármán mid-plane stretching effect. This initial stress is usually due to the mismatch of both thermal expansion coefficient and crystal lattice period between substrate and micro-beam film (Qian et al., 2001). Therefore, this axial force, $N(w)$, can be expressed by (Younis et al., 2006)

$$N(w) = \left[F_r + \frac{EA}{2L}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx\right] \tag{5}$$

where F_r is the initial axial force.