



A reduction model for eigensolutions of damped viscoelastic sandwich structures



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ABSTRACT

The aim of this paper is to develop a reduction method to determine the modal characteristics of viscoelastic sandwich structures. The method is based on the high order Newton algorithm and reduction techniques. Numerical tests have been performed in the case of sandwich beams and cylindrical shells. The comparison of the results obtained by the reduction method with those given by direct simulation shows both a good agreement and a significant reduction in computational cost.

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1. Introduction

To reduce vibrations and noise, viscoelastic materials are often used in many domains (e.g. the aerospace industry, the automobile industry) as passive damping solutions. To maximize this kind of damping, these materials are usually sandwiched between two identical elastic layers. In this configuration, the damping is introduced by an important shear deformation in the viscoelastic central layer. However, the modeling of free vibration problems in viscoelastic sandwich structures involves complex nonlinear eigenvalue problems whose resolution leads to two relevant modal parameters: natural frequencies and loss factors. The nonlinearity is principally due to the dependency of viscoelastic behavior on the frequency (Daya and Potier-Ferry, 2001).

Many investigations have focused on viscoelastic sandwich structures modeling. A review of various theories can be found in Ferreira et al. (2013) and Alvelid (2013). The viscoelastic sandwich eigenvalue resolution leads to nonlinear complex equations. This issue has remained relevant to researchers until now, because of its complexities. Consequently, many research studies can be found in

this field. For example, Daya and Potier-Ferry (2001) developed a continuation method to determine the natural frequencies and the loss factors of viscoelastically damped sandwich structures. They start from the undamped eigenmodes, from which they deduce the viscoelastic modes by using the perturbation technique. Duigou et al. (2003) developed two numerical iterative algorithms for the vibrations of damped sandwich structures. These methods associate homotopy, asymptotic numerical techniques, and Padé approximants. The first one is a sort of high order Newton method; the second one uses a more or less arbitrary matrix. Chen and Chen (2007) analyzed the non-axisymmetric vibration and stability problem of the rotating sandwich plate by using the finite element method, taking into account the effects of transverse shear and rotary inertia. Banerjee et al. (2007) developed an accurate dynamic stiffness matrix for a three-layered sandwich beam of asymmetric cross-section using the Timoshenko beam theory, Hamiltonian mechanics, and symbolic computation. The resulting dynamic stiffness matrix is applied using a Wittrick–Williams algorithm to compute the natural frequencies and mode shapes of some illustrative examples. The authors carried out experimental tests to validate their results. Arikoglu and Ozkol (2010) used a differential transform method in the frequency domain to solve the free vibration equations of a three-layered composite beam with a viscoelastic core. Adhikari and Pascual (2011) proposed a new iterative approach, based on the Biot model, for the calculation of

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eigenvalues of single and multiple degree-of-freedom viscoelastic systems. Damanpack and Khalili (2012) investigated high-order free vibration in three-layered symmetric sandwich beams using the dynamic stiffness method. Attipou et al. (2013) presented a multiscale numerical technique for the free vibration analysis of heterogeneous materials with a constant complex modulus. Researchers have examined the influence of different parameters on the passive damping of the structure. Pawlak and Lewandowski (2013) studied the dynamic characteristics of structures with viscoelastic dampers. They have used the classical and fractional theological models and the continuation method. Many research studies are focused on the parametric study and optimization of viscoelastic structures (Araújo et al., 2012; Sher and Moreira, 2013; Moita et al., 2013).

The issue is that the methods mentioned above could lead to high computational cost in the case of large-scale structures, and only a few studies have focused on cost reduction. Park et al. (1999) used a condensation method to remove only the internal variables of viscoelastic properties. Chen et al. (1999) proposed an iterative reduction method for viscoelastic structures with constant complex modulus. They predicted the damping property in two steps: the first-order asymptotic solution of the nonlinear real eigen equation and the order-reduction-iteration of the nonlinear complex eigen equation. Some authors have used a one-mode Galerkin's procedure to analyze linear and nonlinear vibrations in viscoelastic sandwich structures (Daya et al., 2004; Bilasse et al., 2010, 2011). de Lima et al. (2010) suggested a robust condensation procedure combined with a sub-structuring technique, intended to be used for dealing with large-scale viscoelastically damped structures but only in the case of forced frequency response. Chazot et al. (2011) used a method based on a Padé approximant to reduce the computation time in the dynamic response computation model of multilayered viscoelastic structures. This acceleration technique leads to fast frequency sweep computations, as compared to a standard direct method. More recently, Bilasse and Oguamanam (2013) used real and complex eigenvectors to reduce the forced harmonic equation of large-scale sandwich plates with a viscoelastic core. However, the complex basis requires a nonlinear complex eigenvalue problem resolution.

In this paper, a reduction method for solving the complex nonlinear eigenvalue problem efficiently in the case of large-scale viscoelastic structures is proposed. As in de Lima et al. (2010), the reduction basis is built from real eigenmodes and elastic and viscoelastic stiffness matrices. The reduced complex eigenvalue problem could be resolved by any method cited in the second paragraph above. For instance, the high order Newton algorithm 'HONA' proposed by Duigou et al. (2003) is considered. This high order algorithm is an iterative one based on the coupling of a homotopy transformation and a perturbation technique. This method is efficient but needs triangulation of the stiffness matrix at each iteration. This leads to a significant computational time for large-scale structures. Hence, coupling this method with the reduction technique can reduce the computational cost and memory space considerably and gives an efficient method. The validity and effectiveness of the present method are illustrated in several numerical examples of beams and shell.

2. Problem formulation and reduction model

Assuming the faces and the core are isotropic, the finite element modeling of free vibration in sandwich structures within viscoelastic cores yields a complex nonlinear residual problem (Daya and Potier-Ferry, 2001; Duigou et al., 2003):

$$\mathbf{R}(\mathbf{U}, p) = 0 \tag{1}$$

where $\mathbf{R}(\mathbf{U}, p) = [\mathbf{K}(0) + E(p)\mathbf{K}_v + p^2\mathbf{M}]\mathbf{U}$ represents the residual vector of dimension $[ND]$; $p = i\omega$ is a complex number and ω is the vibration circular frequency; \mathbf{U} is the eigenmode of dimension $[ND]$; \mathbf{M} is the global mass matrix of dimension $[ND \times ND]$; $[\mathbf{K}(0)]$ and $[\mathbf{K}_v]$ are real constant stiffness matrices of dimension $[ND \times ND]$; $E(p)$ is the complex Young's modulus of the viscoelastic core.

As said previously, several methods exist to solve the nonlinear eigenvalue problem (1) directly. So, the j th damped circular frequency ω_j and the j th modal loss factor η_j are obtained using the following formula:

$$\omega_j^2 = \Omega_j^2(1 + i\eta_j) \tag{2}$$

where Ω_j is the j th damped circular frequency (the j th damped frequency $f_j = \Omega_j/2\pi$) and $(i^2 = -1)$.

However, these methods entail a considerable computational cost, especially in the case of large-scale structures, mainly due to the complex tangent matrix triangulations required in nonlinear solver iterations. In order to reduce the computational cost, a reduction technique applied to Eq. (1) has been developed. To do so, the displacement vector is projected onto a small basis:

$$\mathbf{U} = \mathfrak{R}\mathbf{u} \tag{3}$$

where $\mathfrak{R} [ND \times nd]$ is the projection matrix and $\mathbf{u} [nd]$ is the reduced vector.

The reduction model must generate small and limited approximation errors. The system properties, such as stability, must also be preserved. The key point of the reduction technique is the choice of the reduced matrix \mathfrak{R} . This reduction matrix must adequately characterize the nonlinear dynamic response of the structure, and it must be able to approximate the solution in a significant time interval. Moreover, its columns must be linearly independent. Numerous matrices built from (linear eigenvectors, real and imaginary parts of complex eigenvectors, vectors issued from first computation without reduction, etc.) have been tried. Bases built from linear eigenvectors do not give good results even if a large number of modes were taken into account. This is certainly due to the viscoelastic properties of the structure which are not included in the linear modes. The other bases cited above necessitate a large number of columns in order to obtain good results, or they require an update of the base for the computation of each mode. Hence, the base needs a significant computational time to be built and this is not efficient. To reduce the computational time, the base should have a small number of columns, and it should be able to yield good results for different modes.

After trying numerous matrices, we found that the best one (among those used) consists of a matrix built by using linear eigenvectors Φ , the elastic linear matrix $\mathbf{K}(0)$, and the viscoelastic matrix \mathbf{K}_v (de Lima et al., 2010). Then, the projection matrix that is used is given as follows:

$$\mathfrak{R} = [\Phi \quad \mathbf{K}(0)^{-1}\mathbf{K}_v\Phi] \tag{4}$$

Then, if 'Ne' eigenvectors are taken into account in the reduced matrix, the eigenvector matrix is given as follows:

$$\Phi = [\Phi_1, \dots, \Phi_{Ne}] \tag{5}$$

where Φ_i are the eigenvectors and the solutions of the problem:

$$(\mathbf{K}(0) - \omega_i^2\mathbf{M})[\Phi_i] = 0 \tag{6}$$

The reduced matrix of dimension $(nd = 2 \times Ne)$ is given as follows:

$$\mathfrak{R} = [\Phi_1, \dots, \Phi_N, \mathbf{K}(0)^{-1}\mathbf{K}_v\Phi_1, \dots, \mathbf{K}(0)^{-1}\mathbf{K}_v\Phi_N] \tag{7}$$

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