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Elastic thermal stresses in a hollow circular overlay/substrate system

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ABSTRACT

This paper investigates the thermal elastic fields in the hollow circular overlay fully bonded to a rigid substrate, which is subjected to a temperature change. Following our previous work for a solid circular overlay/substrate system (Yuan and Yin, Mech. Res. Commun. 38, 283–287, 2011), this paper presents a closed form approximate solution to the axisymmetric boundary value problem using the plane assumption, whose accuracy is verified by the finite element models. When the inner radius is reduced to zero, the present solution recovers the previous solution. When the outer radius approaches infinite, the solution provides the elastic fields for a tiny hole in the overlay. The effects of thickness and width of the overlay are investigated and discussed. When a circular crack initiates in a solid circular overlay, the fracture energy release rate is investigated. This solution is useful for thermal stress analysis of hollow circular thin film/substrate systems and for fracture analysis of spiral cracking in the similar structures.

1. Introduction

Overlay/substrate systems have been widely used in engineering structures (Leung and Tung, 2006; Timm et al., 2003; Yin et al., 2007a), coatings (Shevchuk and Silberschmidt, 2005; Yin et al., 2007b) as well as material testing and modeling components (Agrawal and Raj, 1989; Hutchinson and Suo, 1992; Porter and Hills, 2001). As energy density increasing in thermal packaging, the thermomechanical behavior of the overlay/substrate systems has attracted significant attention (Suhir, 1989, 1994; Yin et al., 2008). When an overlay/substrate system is subjected to a temperature change, thermal stress is induced due to the mismatch of the coefficients of thermal expansion (CTEs) between the overlay and the substrate and temperature variation in the material. When strain energy reaches a certain level, fractures may initiate to release the stored energy.

Fracture patterns in the thin film have been studied in the past decades. Some unique fracture patterns such as spiral, oscillating and branched fractures have been observed in different overlay/substrate systems (Neda et al., 2002; Cohen et al., 2009; Lazarus and Pauchard, 2011). Yin et al. (2007a) developed a model to investigate transverse crack of asphalt overlay fully bonded to rigid substrate. It has been extended to general materials (Yin et al., 2008), and has been recently applied to investigate the fracture initiation and saturation (Yin, 2010a,b). Neda et al. (2002) modeled the formation mechanism of spiral cracking in drying process using a coarse-grain model and regarded it as logarithmic by fittings of experimental and simulation data. Sendova and Willis (2003) experimentally studied the effects of the film thickness, curing time, and temperature of sol–gel films on the crack geometry, periodicity, and amplitude. Yonezu et al. (2008) investigated the mechanism of formation and propagation of spiral cracking in a thick diamond-like carbon film deposited on a ductile steel substrate. Cohen et al. (2009) evaluated the dependence of crack patterns on the volume contraction and substrate restraint. Lazarus and Pauchard (2011) investigated the effect of the film thickness on crack patterns and obtained craquelures, delamination and spiral cracking morphologies for different film configurations. Nakahara et al. (2011) explored the memory effect of paste to control the crack pattern. Overall, crack patterns in thin films change with the stiffness and strength and fracture toughness of the materials, geometry of the overlay/substrate systems, interface conditions, and loading conditions. To understand the fracture mechanism of the overly/substrate system, stress analysis is needed.

Many researchers have contributed to the investigation of crack propagation. Suo (1989) studied the interactions between singularities and interface cracks and developed the solutions to point force, point moment, edge dislocation and transformation strain spot embedded in bonded elastic blocks of dissimilar materials. Thouless (1995) investigated stress relaxation by cracking and delamination and observed a critical film thickness below which no cracking will occur. For the thermal stress in an overlay/substrate system, many researchers followed the theoretical formulas proposed by Stoney (1909). Unfortunately, this school of semi-empirical approaches cannot be applied under the condition that the substrate is so stiff

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or so thick that no bowing occurs (Suhir, 1994). Suhir (1989, 1994) investigated the magnitude and the distribution of the interface stresses in thermostat-like structures, and evaluated the elastic thermal stresses in a thin film/thick circular substrate. Despite of the large number of the existing models, most of them assumed that stress distributes uniformly or linearly in the thickness direction and the two dimensional (2D) problem can be transferred into a one dimensional (1D) problem (Timm et al., 2003; Suhir, 1994; Xia and Hutchinson, 2000). As a result, they cannot accurately predict the stress distribution because shear stress plays the main role in thermal stress transfer across the interface.

Yin et al. (2007a, 2008) developed a 2D model to predict the stress distribution for rectangular overlay/substrate systems, and extended that model to obtain the explicit solution of the stress distribution in a circular overlay/substrate system (Yuan and Yin, 2011) and layered cylinder material systems (Prieto-Muñoz et al., 2013a,b). Hollow circular overlay/substrate systems have the similar configuration to hollow overlay systems (Zhang et al., 1999; Lungu and Iwasaki, 2002; Dutta et al., 2007). The stress analysis of these material system is important for structure design and fracture analysis. As Part II to our previous paper (Yuan and Yin, 2011), this paper will present a series form solution and a closed form solution of thermal stress in the hollow circular film/substrate system and verify them using the finite element method (FEM) results. The effects of the thickness and the width of the overlay will be discussed.

The remainder of this paper is organized as following: Section 2 presents the basic formulation and derivation of the solution of the problem in the series form. Section 3 introduces a closed form solution by simplifying the solution. In Section 4, the analytical solutions are verified by FEM results, and the analysis about the effects of the overlay thickness and width on the elastic field are presented. Then the fracture energy release rate is investigated when a circular crack initiates at a solid circular overlay at a certain radial position. Finally, some conclusive remarks and future work are provided in Section 5.

2. Basic formulation

Fig. 1 illustrates the top view and the front view of the system: a hollow circular overlay with thickness h, Young's modulus E_1, Poisson's ratio ν_1, the coefficient of thermal expansion (CTE) α_1, inner radius R_1 and outer radius R_2 and a perfectly rigid substrate with CTE α_0. The overlay is fully bonded to the substrate and the whole system is subjected to a temperature change ΔT. Because of the difference of the CTEs of the overlay and the substrate defined as $\bar{\alpha} = \alpha_1 - \alpha_0$, below a thermal stress will be induced in the overlay.

With the assumption that the material is thermoelastically isotropic, the system is axisymmetric. Similar to our previous work (Yuan and Yin, 2011), the coordinate r is set up along the interface between two layers, and the coordinate z is along the axisymmetric axis in the thickness direction. Then the equilibrium equation in the r direction can be written as the following:

$$\sigma_{rr,r} + \tau_{zr,z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \tag{1}$$

Without any constraint applied in the z direction, one can assume that $\sigma_{zz} = 0$. With this assumption, the constitutive law can be written as follows:

$$\begin{cases} \sigma_{rr} = \dfrac{E_1}{1-\nu_1^2}[\varepsilon_{rr} + \nu_1\varepsilon_{\theta\theta} - (1+\nu_1)\bar{\alpha}\Delta T] \\ \sigma_{\theta\theta} = \dfrac{E_1}{1-\nu_1^2}[\nu_1\varepsilon_{rr} + \varepsilon_{\theta\theta} - (1+\nu_1)\bar{\alpha}\Delta T] \\ \tau_{zr} = fracE_1 2(1+\nu_1)\gamma_{zr} \end{cases} \tag{2}$$

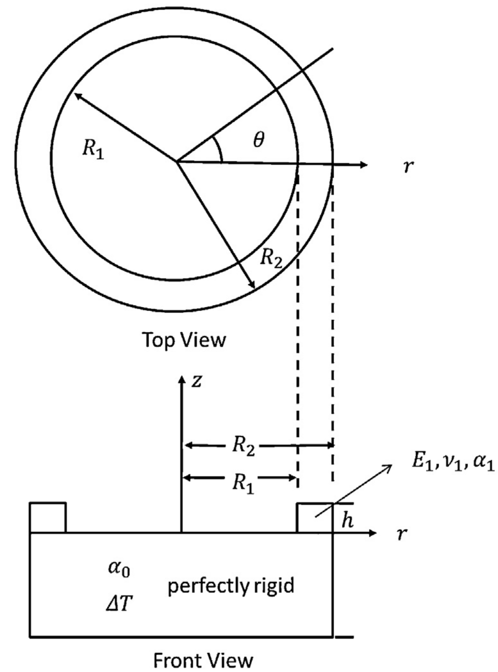

Fig. 1. A hollow circular overlay/substrate system subject to a temperature change.

Because the substrate provides a strong constraint for the overlay, the thermal stress is mainly induced in the plane perpendicular to the z direction. Therefore, one can assume that all the points in a plane normal to the z direction keep in the same plane after the deformation, i.e.:

$$u_z(r,z) = u_z(z) \tag{3}$$

Using Eq. (3), the strain–displacement relation can be written as:

$$\varepsilon_{rr} = u_{r,r}; \quad \varepsilon_{\theta\theta} = \frac{u_r}{r}; \quad \gamma_{zr} = u_{r,z} \tag{4}$$

Inserting Eq. (4) into Eq. (2) and then into Eq. (1) yields the governing equation as follows:

$$\frac{r^2u_{r,rr} + ru_{r,r} - u_r}{r^2} + \frac{1-\nu_1}{2}u_{r,zz} = 0 \tag{5}$$

The above equation can be solved by the method of variable separation. First, one can assume

$$u_r = R(r)Z(z) \tag{6}$$

Substituting Eq. (6) into Eq. (5) yields

$$\frac{r^2R_{,rr} + rR_{,r} - R}{r^2R} = -\frac{1-\nu_1}{2}\frac{Z_{,zz}}{Z} = c^2 \tag{7}$$

where c is a constant that needs to be determined by boundary conditions. Furthermore, one can separate Eq. (7) as

$$(c^2r^2 + 1)R - rR_{,r} - r^2R_{,rr} = 0 \tag{8}$$

$$Z_{,zz} + \frac{2c^2}{1-\nu_1}Z = 0 \tag{9}$$

These two equations can be solved independently. The general solution of Eq. (9) is as

$$Z = A\sin(dz) + B\cos(dz) \tag{10}$$

where A and B are the constants that should be determined by boundary conditions and $d = \sqrt{2/(1-\nu_1)}c$. Compared with the overlay, the substrate is so stiff which can be regarded as a perfectly rigid one. So the mechanical deformation of the substrate is