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The effect of material property grading on the rolling contact stress field

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ABSTRACT

This paper investigates the subsurface stress field induced by a rigid cylinder rolling over a functionally graded coating-substrate system. The Fourier transform is employed to extract the stress components within the graded coating and the homogeneous substrate. The distributions of the stresses are given through the depth and along the coating–substrate interface. The contour plots of normalized Von Mises stresses are provided as well. The results indicate that continuous variation of the shear modulus substantially reduces the difference between the in-plane stresses along the interface. Also, the softening coating leads to the minimum value of the stress concentration near the contact surface.

1. Introduction

The functionally graded materials (FGMs) with intentionally variable material properties offer several advantages over their conventional homogeneous counterparts. For example, Suresh et al. (1999) proved that controlled gradients of the elastic modulus can substantially increase the surface resistance to the contact damages. Nowadays, the graded coatings are widely used in practical applications since they are capable to resist severe stress and temperature gradients as well as corrosive and abrasive conditions (Suresh, 2001).

As a result, comprehensive studies have been carried out to explore the effect of graded coatings on the behavior of several contact problems. Giannakopoulos and Suresh (1997) assumed both the exponential and the power law functions for the variation of the elastic modulus through the half-plane and formulated the point force indentation of the gradient materials in an analytical framework. Ke and Wang (2006) utilized a series of continuous but piecewise linear curves to simulate the arbitrary variation of the elastic modulus within the graded materials and analyzed the two-dimensional sliding contact problem of a graded coating–substrate system. Assuming an exponential variation of the shear modulus, El-Borgi et al. (2006) investigated the receding contact problem of a graded layer resting on a homogeneous half-space. Using a similar exponential model, Choi and Paulino (2010) examined the interfacial cracking of a graded interlayer under the effect of a surface sliding punch. Dag et al. (2012) conducted the fracture analysis of an exponentially graded coating containing a surface crack subjected to the contact stresses. Chidlow et al. (2011) employed a Fourier series based solution to determine the stress field and the deformation throughout an FGM coating–substrate system induced by a known contact pressure.

Recently, Guler et al. (2012) evaluated the surface tractions ($p(x)$ and $q(x)$) induced by a rigid cylinder, rolling over an FGM coating bonded to a homogeneous substrate (see Fig. 1). On the other hand, the precise design and manufacturing process require better intuition about the stress state beneath the contact surface, the regions of stress concentration and the distribution of the flexural stress over the coating–substrate interface. In this paper, the subsurface stresses are linked to the surface tractions by means of Fourier transform technique. All the stress components are extracted within the coatings and the substrate as well. Finally, some important numerical results are provided.

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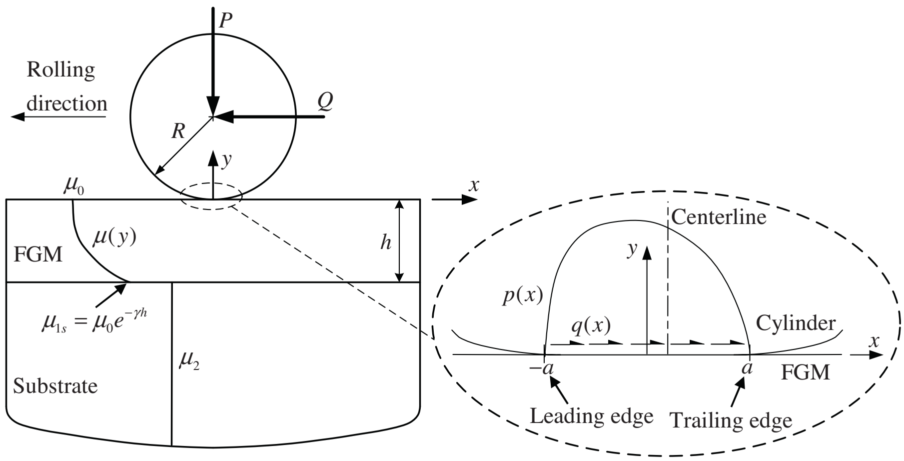

Fig. 1. Geometry of the rolling contact between a rigid cylinder and the coating-substrate system.

2. Formulation of the problem

The problem geometry is given in Fig. 1. A rigid cylinder of a radius R rolls over a graded coating of a thickness h under the action of normal, P, and horizontal, Q, external forces. It is assumed that the shear modulus continuously varies along the coating thickness such that:

$$\mu(y) = \begin{cases} \mu_0 e^{\gamma y}, & -h \le y \le 0, \\ \mu_2, & y < -h, \end{cases} \tag{1}$$

Note that μ_0 is the shear modulus value at the surface of the coating and μ_2 is the shear modulus of the homogeneous substrate (see Fig. 1). Also, we define:

$$\gamma h = -\ln(\Gamma), \tag{2}$$

$$\Gamma = \frac{\mu_{1s}}{\mu_0}, \tag{3}$$

$$\chi = \frac{\mu_{1s}}{\mu_2}, \tag{4}$$

where Γ is known as the stiffness ratio, χ is the interface shear modulus ratio and μ_{1s} is the shear modulus value at $y = -h$. Note that the Poisson's ratio, ν, remains constant throughout the system. Using the singular integral equations approach, Guler et al. (2012) formulated the rolling contact problem of the graded coatings which renders the contact surface tractions. Here, the stress components within the body are related to the surface tractions by means of Fourier transform. Referring to the Hooke's law, the stress components within the graded coating are given as (Guler and Erdogan, 2004):

$$\sigma_{xx}(x, y) = \frac{\mu_0 e^{\gamma y}}{\kappa - 1}\left\{(\kappa+1)\frac{\partial u}{\partial x} + (3-\kappa)\frac{\partial v}{\partial y}\right\}, \tag{5}$$

$$\sigma_{yy}(x, y) = \frac{\mu_0 e^{\gamma y}}{\kappa - 1}\left\{(3-\kappa)\frac{\partial u}{\partial x} + (\kappa+1)\frac{\partial v}{\partial y}\right\}, \tag{6}$$

$$\sigma_{xy}(x, y) = \mu_0 e^{\gamma y}\left\{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right\}, \tag{7}$$

where $u(x, y)$ and $v(x, y)$ are the horizontal and the vertical displacement components within the coating, respectively, and $\kappa = 3 - 4\nu$ is the Kolosov's constant for the plane strain conditions. Applying the Fourier transform in x direction, it can be shown that (Guler and Erdogan, 2004):

$$2\pi\mu_0 \frac{\partial}{\partial x} v(x, y) = \int_{-a}^{a} K_{31}(x, y, w)\sigma(w)dw + \int_{-a}^{a} K_{32}(x, y, w)\tau(w)dw, \tag{8}$$

$$2\pi\mu_0 \frac{\partial}{\partial x} u(x, y) = \int_{-a}^{a} K_{41}(x, y, w)\tau(w)dw + \int_{-a}^{a} K_{42}(x, y, w)\sigma(w)dw, \tag{9}$$

where $\sigma(x) = \sigma_{yy}(x, 0)$ and $\tau(x) = \sigma_{xy}(x, 0)$ are the contact surface stresses and:

$$K_{mn}(x, y, w) = \int_{-\infty}^{\infty} h_{mn}(\alpha, y)e^{-i\alpha(w-x)}d\alpha, \quad m = 3, 4; \quad n = 1, 2. \tag{10}$$

Note that the functions $h_{mn}(\alpha, y)$, $m = 3, 4$; $n = 1, 2$ were given by Guler (2009). Utilizing Eqs. (8) and (9) and considering Eq. (10), the displacement gradients in y direction are evaluated as:

$$2\pi\mu_0 \frac{\partial}{\partial y} v(x, y) = \int_{-a}^{a} K'_{31}(x, y, w)\sigma(w)dw + \int_{-a}^{a} K'_{32}(x, y, w)\tau(w)dw, \tag{11}$$

$$2\pi\mu_0 \frac{\partial}{\partial y} u(x, y) = \int_{-a}^{a} K'_{41}(x, y, w)\tau(w)dw + \int_{-a}^{a} K'_{42}(x, y, w)\sigma(w)dw, \tag{12}$$

where:

$$K'_{mn}(x, y, w) = \int_{-\infty}^{\infty} \frac{1}{i\alpha}\frac{\partial}{\partial y} h_{mn}(\alpha, y)e^{-i\alpha(w-x)}d\alpha, \quad m = 3, 4; \quad n = 1, 2. \tag{13}$$

Substituting Eqs. (8), (9) and (11), (12) back into Eqs. (5)–(7) and rearranging, the stress components within the graded coating are