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Heat conduction analysis of laminated shells by a sampling surfaces method

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ABSTRACT

A paper focuses on the use of the method of sampling surfaces (SaS) for the exact three-dimensional (3D) heat conduction analysis of laminated orthotropic and anisotropic shells. This method is based on selecting inside the nth layer I_n not equally spaced SaS parallel to the middle surface of the shell in order to choose the temperatures of these surfaces as basic variables. Such an idea permits the representation of the proposed thermal laminated shell formulation in a very compact form. The SaS are located inside each layer at Chebyshev polynomial nodes that improves the convergence of the SaS method significantly. As a result, the SaS method can be applied efficiently to exact 3D solutions of the steady-state heat conduction problem for cross-ply and angle-ply composite shells with a specified accuracy using a sufficient number of SaS.

1. Introduction

Nowadays, it is well established that for the accurate analysis of quasi-static thermoelasticity and thermopiezoelectricity problems for orthotropic and anisotropic laminated plates and shells it is necessary to solve the Fourier heat conduction equation because no prescribed through-thickness temperature distributions can be utilized (see, e.g. Tungikar and Rao, 1994; Savoia and Reddy, 1995; Soldatos and Ye, 1995; Kapuria et al., 1997; Tauchert et al., 2000; Vel and Batra, 2001, 2003; Brischetto, 2009; Brischetto and Carrera, 2011). This means that any algorithm for the numerical solution of the Fourier heat conduction equation must be incorporated in advanced computational models developed for the thermal stress analysis of laminated composite shells of arbitrary geometry and general layup configurations (Noor and Burton, 1992; Reddy, 2004). However, it is not a simple task because we deal here with the partial differential equation with variable coefficients depending on the thickness coordinate and many nodes in the thickness direction can be required to find the reliable results for thick shells.

To solve such a problem efficiently, we invoke the method of sampling surfaces (SaS) developed recently for the exact three-dimensional (3D) stress analysis of elastic and piezoelectric laminated plates (Kulikov and Plotnikova, 2012b, 2013a) and shells (Kulikov and Plotnikova, 2013b, 2013c). As SaS denoted by $\Omega^{(n)1}$, $\Omega^{(n)2}$, …, $\Omega^{(n)I_n}$, we select outer surfaces and any inner surfaces inside the nth layer in order to introduce the temperatures $T^{(n)1}$, $T^{(n)2}$, …, $T^{(n)I_n}$ of these surfaces as basic shell variables, where I_n is the number of SaS chosen for each layer ($I_n \geq 3$). This choice of temperatures with the consequent use of Lagrange polynomials of degree $I_n - 1$ in the thickness direction for each layer permits the representation of the governing equations in a very compact form. It is important that the developed approach with the arbitrary number of *equally spaced* SaS inside the shell body (Kulikov and Plotnikova, 2011a, 2011b) does not work properly with Lagrange polynomials of high degree because the Runge's phenomenon can occur, which yields the wild oscillation at the edges of the interval when the user deals with any specific functions. If the number of *equispaced* nodes is increased then the oscillations become even larger. Fortunately, the use of Chebyshev polynomial nodes (Kulikov and Plotnikova, 2012b, 2013c) can help to improve significantly the behavior of Lagrange polynomials of high degree for which the error will go to zero as $I_n \to \infty$.

An idea of using the SaS can be traced back to contributions of Kulikov (2001), and Kulikov and Carrera (2008) in which three, four and five *equally spaced* SaS are utilized. It is necessary to mention that in a layerwise differential quadrature (LWDQ) analysis (Liew et al., 2002, 2003; Zhang et al., 2003; Malekzadeh, 2009; Malekzadeh et al., 2008; Setoodeh et al., 2011) the nodal surfaces inside the mathematical layer are introduced following the general layerwise concept (Reddy, 2004; Carrera, 2003). The main difference consists in the lack of possibility to employ the Lagrange polynomials of high degree in the thickness direction. This is due to the fact that in a conventional LWDQ formulation only *equally spaced* nodal surfaces inside the mathematical layer are admissible

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with the use of the simplest Lagrange polynomials of first and second orders. The next feature of the LWDQ method is that each nodal surface is discretized into a set of grid points in both plane directions. This means that one deals here with a numerical technique. On the contrary, the SaS method can be used efficiently for analytical developments (Kulikov and Plotnikova, 2012b, 2013c) and numerical implementations in strong and weak forms (Kulikov and Plotnikova, 2011a, 2011b, 2012a) as well.

The same concerns the finite layer method (Cheung and Jiang, 2001), which is the most efficient semi-analytical method for the 3D analysis of simply supported plates and shells (Akhras and Li, 2007, 2010; Wu and Li, 2010; Wu and Chang, 2012; Wu and Kuo, 2013). In this method, the shell is divided into a number of finite layers. Within each finite layer, the trigonometric functions are employed for in-plane interpolations of displacements in a displacement-based formulation (Wu and Kuo, 2013) and additionally transverse stresses in a mixed formulation (Wu and Chang, 2012), whereas the lower-order Lagrange polynomials with *equispaced* nodal points are accepted for the interpolation in the thickness direction, i.e. the h-refinement is adopted. Thus, the difference between the SaS method and the finite layer method consists in the following: the p-refinement is used in the former, while the h-refinement is used in the latter. Wu and his coauthors showed convincingly that the finite layer method with *equally spaced* nodal surfaces yields good predictions of the mechanical behavior of composite plates and shells. However, the 3D solutions derived are approximate. To obtain the exact 3D solutions, the p-refinement should be invoked. As pointed out earlier, the SaS method utilizes the Lagrange polynomials of high degree with Chebyshev polynomial nodes that allows one to minimize uniformly the error due to Lagrange interpolation. This fact gives an opportunity to find the exact 3D solutions for thermal laminated composite shells with a prescribed accuracy employing the sufficiently large number of SaS.

The present paper is intended to show that the SaS method can be applied efficiently to the exact solutions of some 3D steady-state problems of the heat conduction theory of laminated composite shells. The authors restrict themselves to finding five right digits in all examples presented except for a Section 4 with the results of the convergence study. The better accuracy is possible but requires more SaS inside each layer to be taken.

2. Description of temperature field

Consider a thick laminated shell of the thickness h. Let the middle surface Ω be described by orthogonal curvilinear coordinates θ_1 and θ_2, which are referred to the lines of principal curvatures of its surface. The coordinate θ_3 is oriented along the normal to the middle surface. Introduce the following notations: A_α are the coefficients of the first fundamental form of the middle surface; k_α are the principal curvatures of the middle surface; $\theta_3^{(n)i_n}$ are the transverse coordinates of SaS of the nth layer given by

$$\theta_3^{(n)1} = \theta_3^{[n-1]}, \quad \theta_3^{(n)I_n} = \theta_3^{[n]},$$
$$\theta_3^{(n)m_n} = \frac{1}{2}(\theta_3^{[n-1]} + \theta_3^{[n]}) - \frac{1}{2}h_n \cos\left(\pi \frac{2m_n - 3}{2(I_n - 2)}\right), \tag{1}$$

where $\theta_3^{[n-1]}$ and $\theta_3^{[n]}$ are the transverse coordinates of layer interfaces $\Omega^{[n-1]}$ and $\Omega^{[n]}$ depicted in Fig. 1; $h_n = \theta_3^{[n]} - \theta_3^{[n-1]}$ is the thickness of the nth layer; I_n is the number of SaS corresponding to the nth layer; the index m_n identifies the belonging of any quantity to the inner SaS of the nth layer and runs from 2 to $I_n - 1$, whereas the indices i_n, j_n, k_n to be introduced later for describing all SaS of the nth layer run from 1 to I_n; the index n identifies the belonging of any quantity to the nth layer and runs from 1 to N, where N is the total number of layers. Besides, the tensorial indices i, j range from 1 to 3 and Greek indices α, β range from 1 to 2.

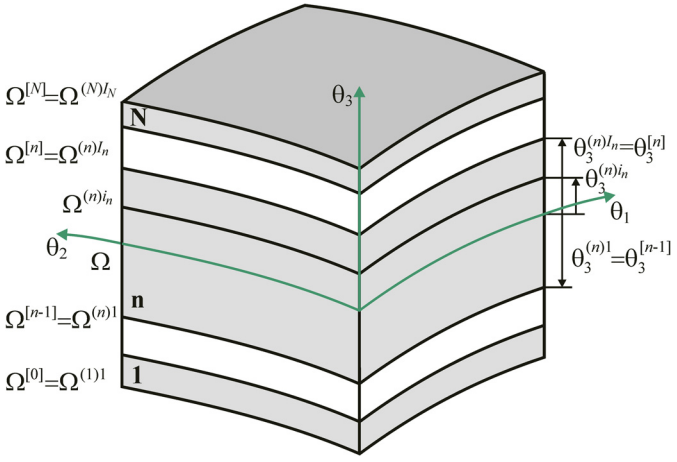

Fig. 1. Geometry of the laminated shell.

Remark 1. The transverse coordinates of inner SaS (1) coincide with the coordinates of Chebyshev polynomial nodes (Burden and Faires, 2010). This fact has a great meaning for a convergence of the SaS method (see, e.g. Kulikov and Plotnikova, 2012c).

The relation between the temperature T and the temperature gradient $\mathbf{\Gamma}$ is given by

$$\mathbf{\Gamma} = \nabla T. \tag{2}$$

In a component form, it can be written as

$$\Gamma_\alpha = \frac{1}{A_\alpha c_\alpha} T_{,\alpha}, \quad \Gamma_3 = T_{,3}, \tag{3}$$

where $c_\alpha = 1 + k_\alpha \theta_3$ are the components of the shifter tensor; the symbol $(\dots)_{,i}$ stands for the partial derivatives with respect to coordinates θ_i.

We start now with the first assumption of the proposed thermal laminated shell formulation. Let us assume that the temperature and temperature gradient fields are distributed through the thickness of the nth layer as follows:

$$T^{(n)} = \sum_{i_n} L^{(n)i_n} T^{(n)i_n}, \quad \theta_3^{[n-1]} \le \theta_3 \le \theta_3^{[n]}, \tag{4}$$

$$\Gamma_i^{(n)} = \sum_{i_n} L^{(n)i_n} \Gamma_i^{(n)i_n}, \quad \theta_3^{[n-1]} \le \theta_3 \le \theta_3^{[n]}, \tag{5}$$

where $T^{(n)i_n}(\theta_1, \theta_2)$ are the temperatures of SaS $\Omega^{(n)i_n}$ of the nth layer; $\Gamma_i^{(n)i_n}(\theta_1, \theta_2)$ are the components of the temperature gradient at the same SaS; $L^{(n)i_n}(\theta_3)$ are the Lagrange polynomials of degree $I_n - 1$ defined as

$$T^{(n)i_n} = T(\theta_3^{(n)i_n}), \tag{6}$$

$$\Gamma_i^{(n)i_n} = \Gamma_i(\theta_3^{(n)i_n}), \tag{7}$$

$$L^{(n)i_n} = \prod_{j_n \ne i_n} \frac{\theta_3 - \theta_3^{(n)j_n}}{\theta_3^{(n)i_n} - \theta_3^{(n)j_n}}. \tag{8}$$

The use of relations (3), (4), (6) and (7) yields

$$\Gamma_\alpha^{(n)i_n} = \frac{1}{A_\alpha c_\alpha^{(n)i_n}} T_{,\alpha}^{(n)i_n}, \tag{9}$$

$$\Gamma_3^{(n)i_n} = \sum_{j_n} M^{(n)j_n}(\theta_3^{(n)i_n}) T^{(n)j_n}, \tag{10}$$