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On the resonance frequencies of a membrane of a dielectric elastomer

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ABSTRACT

The resonance frequencies of a pre-stretched circular membrane of a dielectric elastomer are investigated. The resonance frequencies increase with mode and thickness of the membrane, but they decrease in air from those in vacuum due to the added mass of air. The damping of air is low and has negligible effect on the frequencies; however, it helps to reduce the amplitude of vibration, comparing with that in the vacuum. The frequencies decrease with an increase of the applied voltage, the mass of the electrodes, and the radius of the circular membrane. The effect of applied pressure on the resonance frequencies of the membrane is not significant.

1. Introduction

Dielectric elastomers are smart and electroactive materials which deform due to the applied voltage. The membrane of dielectric elastomers has many applications (Heydt et al., 2000; Dubois et al., 2008; Koh et al., 2009; McKay et al., 2010; Zhu et al., 2010; Fox and Goulbourne, 2008, 2009; Chakravarty and Albertani, 2011a, 2012) that include in robotics, cardiac membrane pump, an artificial bicep for orthotic and prosthetic technology, programmable haptic surfaces, loud speakers, micro air vehicles, energy harvesting, active noise control, sensing, and RF filtering where the resonance frequencies may need to be very precisely controlled. It is very important to investigate the resonance frequencies of the dielectric elastomer's membrane for these types of device design and optimization. Recently, many people are investigating the mechanical properties of the membrane of dielectric elastomers due to its diverse applications, ease of fabrication, low cost, and high deformability. Quasi-static experiments were conducted for estimating the hyperelastic material properties of the membrane (Fox and Goulbourne, 2008, 2009; Chakravarty and Albertani, 2011b). Experimental vibration analysis of the membrane was presented by Fox and Goulbourne (2008, 2009), Chakravarty and Albertani (2011a, 2012), and Jenkins and Korde (2006). It is found from the experimental results that the resonance frequencies and amplitude of vibration of a membrane change due to the added mass and damping of surrounding air (Chakravarty and Albertani, 2011a, 2012).

The nonlinear vibration characteristics of the membrane were investigated by a few researchers (Zhu et al., 2010; Fox and Goulbourne, 2008, 2009; Chakravarty and Albertani, 2011a, 2012; Gonçalves et al., 2009; Jenkins and Leonard, 1991; Mockensturm and Goulbourne, 2006; Tuzel and Erbay, 2004; Jiang et al., 1992), where hyperelastic material models were considered. There are several hyperelastic material models available, such as Mooney–Rivlin, neo-Hookean, Ogden, Yeoh, and Arruda–Boyce hyperelastic material models (Chakravarty and Albertani, 2011b; Mooney, 1940; Rivlin, 1948; Treloar, 1944; Boyce and Arruda, 2000; Yeoh, 1993). Gonçalves et al. (2009) developed the analytical and finite element (FE) models for examining the dynamic behavior of a radially pre-stretched circular membrane without the effect of voltage and the added mass of surrounding fluid. Zhu et al. (2010) investigated the resonant behavior of a pre-stretched membrane of a dielectric elastomer without the effect of added mass and damping of surrounding air.

This paper presents the analytical and FE models for investigating the vibration characteristics of a radially pre-stretched circular membrane of a dielectric elastomer. The effect of voltage, added mass, damping, and pressure on the resonance frequencies of the membrane is examined. The variations of resonance frequencies with the mass of two electrodes, thickness and radius of the membrane are also investigated.

2. Analytical model

For the analytical solution, a flat circular membrane specimen of initial radius of R_i and initial thickness of h_i is radially pre-stretched and clamped at the boundary with a rigid ring of radius of R_f (stretch

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ratio $\Re = R_f/R_i$). Thin carbon grease electrodes are attached to the top and bottom surfaces of the membrane for applying the DC voltage of E. The Mooney–Rivlin hyperelastic material model is considered for the membrane and the Mooney–Rivlin hyperelastic material parameters are C_1 and C_2. The out-of-plane deformation of a point at an arbitrary location $X(r, \theta)$ on the membrane is $w(r, \theta, t)$ due to vibration. The equation of motion can be expressed in cylindrical coordinate system as (Chakravarty, 2013).

$$\frac{\partial^2 w}{\partial t^2} = c_s{}^2\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right) \tag{1}$$

for $0 \le r < R_f$, $0 \le \theta \le 2\pi$ and $w(R_f, \theta, t) = 0$, where $c_s{}^2 = \left\{[2(C_1 + C_2) + 2C_2(\Re^2 - 1)](1 - 1/\Re^6) - \varepsilon[E/(h_f\Re)^2\right\}/\rho$, ε is the permittivity of the membrane, h_f is the thickness of the pre-stretched membrane ($h_f = h_i/\Re^2$), and ρ is the density of the membrane-electrode material system. The principal in-plane pre-stresses of the membrane are equal to $[2(C_1+C_2)+2C_2(\Re^2-1)](\Re^2-1/\Re^4)-\varepsilon E^2$. The free vibration modes $w_{mn}(r, \theta, t)$ are found by solving the above equation (Chakravarty, 2013), based on the required boundary and continuity conditions, i.e.

$$w_{mn}(r, \theta, t) = A_{mn} J_m\left(\eta_{mn}\frac{r}{R_i}\right)\cos(m\theta)\cos(\omega_{mn}t) \tag{2}$$

where A_{mn} is an arbitrary constant and known as the modal amplitude of vibration; J_m is the m-th order ($m = 0, 1, 2, \ldots, \infty$) Bessel function of the first kind; η_{mn} are the zeros of the Bessel function, J_m ($n = 1, 2, 3, \ldots, \infty$); ω_{mn} is the circular resonance frequency of the (m, n) mode of vibration and can be calculated from the following equation.

$$\omega_{mn} = \eta_{mm}\sqrt{\frac{[2(C_1 + C_2) + 2C_2(\Re^2 - 1)](1 - 1/\Re^6) - \varepsilon[E/(h_f\Re)]^2}{\rho R_i{}^2}} \tag{3}$$

The surrounding air exerts force and opposes the movement when the membrane vibrates in air. The added mass is the mass of the surrounding air that is required to accelerate for the acceleration of the membrane. As a result, the resonance frequencies of the membrane decrease due to the effect of added mass. The added mass depends on the geometry of the membrane and the density of air. The added mass m_a of the circular membrane is $m_a = (8/3)R_f^3\rho_a$ when it vibrates in air (Chakravarty and Albertani, 2011a, 2012; Azuma, 2006), where R_f and ρ_a are the radius of the circular membrane and density of surrounding air, respectively.

3. Results and discussion

For this paper, a circular membrane specimen of the dielectric elastomer VHB 4910 is radially pre-stretched at stretch ratio of 3 and attached to a circular rigid ring of radius of 50 mm. The initial thickness, density, and dielectric constant of the membrane are 1.0 mm, 960 kg/m^3 and 4.55, respectively (Zhu et al., 2010). The Mooney–Rivlin hyperelastic material parameters of the membrane are C_1 = 16 kPa and C_2 = 7.3 kPa (Fox and Goulbourne, 2008). The mass of the electrodes is considered 4 times higher than that of the membrane specimen (Zhu et al., 2010). The stretch ratio, geometry (radius and thickness) of the membrane, and the mass of the electrodes remain same as above for computing the resonance frequencies, excluding where they are mentioned in the following paragraphs.

An FE model is also developed for investigating the vibration characteristics of the pre-stretched circular membrane, clamped at the boundary, using the FE analysis software, Abaqus 6.10® (SIMULIA, Providence, Rhode Island 02909, 2010). The FE model is developed considering the following three steps.

(1) Calculate the principal in-plane pre-stresses of the membrane including the effect of the electrostatic force.
(2) Define the pre-stresses of the membrane as input data and run the linear and nonlinear static analysis.
(3) Run the resonance frequency analysis of the membrane including the effect of damping, added mass of air, and mass of the electrodes.

M3D6 (6-node quadratic triangular membrane) type of elements are selected for the membrane. The effect of added mass of surrounding air is included in the FE model. The added mass of surrounding air is added with the actual mass of the membrane in the FE model. Rayleigh damping is considered and the damping is provided in the FE model as Rayleigh damping parameters (Chakravarty and Albertani, 2011a, 2012; Cook et al., 1989). The convergence of the resonance frequencies of the membrane is studied and it is found that the frequencies converge on the order of 1000 degrees of freedom. It is also found that the FE model correlates well with the experimental data, reported by Chakravarty and Albertani (2012).

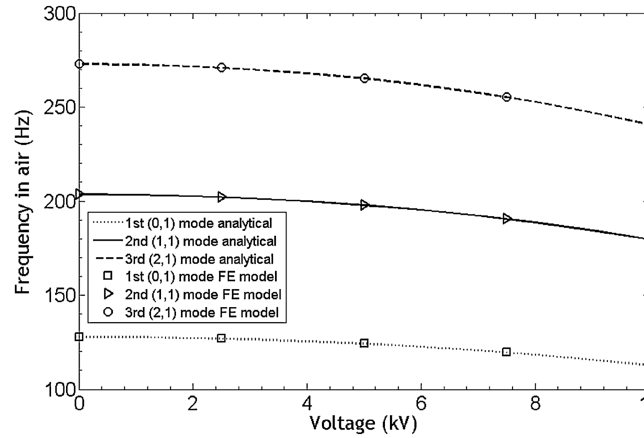

Fig. 1. Resonance frequencies vs. voltage plots for the membrane specimen in air.

Resonance frequencies of the pre-stretched circular membrane specimen at different voltages are computed in vacuum and in air (at atmospheric pressure) by using both the analytical and FE models and the first three mode resonance frequencies in air are shown in Fig. 1. A good correlation is found among the resonance frequencies computed by the analytical and FE models (vary less than 0.05%). Fig. 1 depicts that resonance frequencies increase with mode. It is well known that the first (0, 1) mode frequency means the fundamental (lowest) frequency of vibration. So the second (1, 1) mode frequency is higher than that of the first mode and so on. Resonance frequencies of the membrane specimen decrease 4.54% in air from those in vacuum due to the added mass of surrounding air. The frequencies decrease because the mass of the membrane increases due to the added mass of the surrounding air, although the stiffness of the membrane remains constant. Fig. 1 also shows that resonance frequencies decrease with voltage. The applied voltage helps to reduce the internal stress of the pre-stretched membrane (Zhu et al., 2010). As a result, the stiffness of the membrane decreases which leads to the decrease of the resonance frequencies due to an increase of applied voltage.

The variation of the first (0, 1) mode resonance frequency with the ratio of the masses of the electrode and the membrane specimen at three different voltages (0, 5, and 10 kV) in air is shown in Fig. 2. The ratio is calculated by dividing the mass of the electrodes