Mechanics Research Communications 53 (2013) 47–52

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Identification of a fault zone ahead of the tunnel excavation face using the extended Kalman filter

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ARTICLE INFO

Article history:
Received 22 November 2012
Received in revised form 11 July 2013
Accepted 6 August 2013
Available online 16 August 2013

Keywords:
Extended Kalman filter
Parameter identification
Fault zone
Back analysis
Tunnel excavation

ABSTRACT

Simulation of mechanized tunneling and on-site excavation require very good knowledge of the geomechanical and material properties. Identification of the material must be fast and continuously performed during tunnel excavation for the best possible strategies for advancing the tunnel boring machine. We present in this work the use of the extended Kalman filter (EKF) for identification of the inclined fault zone ahead of the face. The EKF showed fast and stable convergence of the model parameters under study. In comparison with the particle swarm optimization technique applied to the same back analysis problem, faster convergence of the identified parameters as well as high robustness with respect to the choice of the initial parameter values have been observed.

1. Introduction

The back analysis procedures require reliable and sufficient measurement data, a robust numerical model and an efficient method for the solution of the inverse (back analysis) problem (Gioda and Sakurai, 1987). However *in situ* measurements are susceptible to uncertainties due to instrument inaccuracies, environment and human handling. Furthermore, for geotechnical problems, the back analysis employs direct approach based on iterative solution of a forward problem by means of a numerical approximation (*e.g.* finite element analysis) and therefore the exact model response is basically unknown (Gioda and Sakurai, 1987; Gioda and Locatelli, 1999; Schanz et al., 2006).

The Kalman filter method was developed by *Rudolf E. Kalman* in early 1960s (Kalman, 1960). Since that time it has been widely employed in signal processing, mechanical systems, etc. as a very successful method for state estimation. Since late 1980s, some applications of the Kalman filter method have also arisen in the field of geotechnical engineering for parameter identification of elastic and ideally elastoplastic material parameters (Murakami and Hasegawa, 1988; Murakami, 1991; Hoshiya and Suto, 1993), *in situ* stresses in rock mass (Yang et al., 2011). According to the categorization of back analysis methods introduced by Gioda and Sakurai (1987), the Kalman filter belongs to the probabilistic back analysis approach. In this work, we present the use of the EKF to estimate the location, the thickness, and the elastic modulus of the fault zone ahead of the tunnel excavation face using a set of settlement and horizontal displacement data at observation positions around the tunnel. To test the efficiency of the method, we use a set of synthetic measurement data calculated by a finite element model as measurement data. Afterwards, we applied the Kalman filter method to back analyse the parameters of interest.

The Kalman filter method is based on the recursive least squares estimation method adapted to time updates and observation updates of the mean state and its covariance, taking into account uncertainties of system modeling and measurements (Kalman, 1960; Gelb, 1974). It starts from *a priori* estimate and utilizes a set of observation data to calculate *a posteriori* estimate. The sequence is repeated until convergence has been ascertained. One of the main features of the Kalman filter method is that the estimation of parameters at any time instant is accompanied by relevant covariance matrices, which represent the uncertainty of the estimated parameters, as well as of the process noise and observation noise. Due to this property of the Kalman filter, it is classified as a statistical identification approach. Murakami (1991) has pointed out that the Kalman filter and the Bayesian estimator are equivalent. For the application of Bayes' approach in parameter estimation of geotechnical models the readers may refer to Cividini et al. (1983).

The Kalman filter is the best unbiased filter for state estimation of linear systems. However, since formulations of the parameter identification (or the so-called back analysis) mostly result in

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0093-6413/$ – see front matter
http://dx.doi.org/10.1016/j.mechrescom.2013.08.003

nonlinear systems, an adequate formulation of the Kalman filter is required as well. In this paper, the extended Kalman filer (EKF), which represents a version of the Kalman filter for nonlinear systems linearized around the current state, will be employed.

In the next section of this paper, the EKF for parameter identification problem will be formulated with focuses on calculation of the sensitivity matrix and adjustment of the error covariance matrix for obtaining fast convergence. After that, we present and discuss the results of application of the EKF for identification of an inclined fault zone ahead of the tunnel face during tunnel excavation. Finally, we conclude the work and give remarks on implementation and adjustment of the EKF for parameter identification problems in geomechanics.

2. The extended Kalman filter (EKF)

A state-space formulation of a general mechanical system can be represented for the problem of parameter identification in the following form:

$$\mathbf{x}(k+1) = \mathbf{x}(k) + \mathbf{w}(k), \tag{1a}$$

$$\mathbf{y}(k) = \mathbf{h}(\mathbf{x}(k)) + \mathbf{v}(k), \tag{1b}$$

where components of the vector $\mathbf{x}$ are the model parameters that require identification. For the parameter identification, the state transition equation, Eq. (1a), is represented as stationary with additive noise $\mathbf{w}$ at imaginary time step t_k in the identification process. The formulation of the state space dynamic system representation in discrete-time form, which is well known in the control system theory:

$$\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}\mathbf{u}(k) + \mathbf{w}(k) \tag{2}$$

is reformulated accordingly for a stationary estimation problem with Kalman filter in order to obtain Eq. (1a), so that the state matrix $\mathbf{\Phi}$ remains equal to identity matrix, and no control input $\mathbf{u}$ is employed. The elements of the state vector $\mathbf{x}$ become in this formulation the parameters to be estimated, but their evolution throughout the estimation algorithm may be considered as a discrete-time stepwise change till final convergence to the real value has been achieved, and therefore with respect to the number of iterations k they are "dynamically" changing in discrete time steps. Therefore we refer to $\mathbf{x}$ as the state vector for the reason that it represents the state of the parameters set in this employed recursive estimation algorithm. The measurements, or observations, required for the state estimation are related with the parameters to be estimated by appropriate constitutive law, expressed in term of the output equation in the state space formulation adopted in the control systems representation. The choice of the type of measurements depends on the particular geotechnical problem and the available measurement tools. For more details on the measurement (output) equation formulation readers are referred to Nestorović and Nguyen (2013).

With respect to the parameter set represented by the state vector $\mathbf{x}$, modeling data $\mathbf{y}$ can be obtained at the selected observation positions through the use of numerical modeling function $\mathbf{h}(\cdot)$ as presented in Eq. (1b). The fact that the model responses are nonlinearly related to the model parameters even when the geotechnical problem is linear is given in Hoshiya and Suto (1993). Additive uncertainty $\mathbf{v}$ associated with modeling function is to represent the inaccuracies caused by numerical approximation that is used to solve the forward problem. Both $\mathbf{w}$ and $\mathbf{v}$ are assumed to have Gaussian distribution with zero means and covariance matrices $\mathbf{Q}$ and $\mathbf{R}$ respectively. Covariance matrices $\mathbf{Q}$ and $\mathbf{R}$ are mathematically described as

$$E[\mathbf{w}(k)\mathbf{w}^T(k)] = \mathbf{Q}; \quad E[\mathbf{v}(k)\mathbf{v}^T(k)] = \mathbf{R}. \tag{3}$$

In order to explain the recursive handling between time updates and measurement updates of the Kalman filter we introduce definition of *a priori estimate*, the estimate of the state before observation data are available, and *a posteriori estimate*, the estimate of the state after observation data are available. In mathematical expressions, *a priori* state estimate $\hat{\mathbf{x}}(k+1|k)$ and its covariance matrix $\mathbf{P}(k+1|k)$ at time t_{k+1} are estimated as follows given the observation data up to time t_k:

$$\hat{\mathbf{x}}(k+1|k) = E[\mathbf{x}(k+1)|\mathbf{y}(1), \mathbf{y}(2), \ldots, \mathbf{y}(k)],$$
$$\mathbf{P}(k+1|k) = E[\{\mathbf{x}(k+1) - \hat{\mathbf{x}}(k+1|k)\}\{\mathbf{x}(k+1) - \hat{\mathbf{x}}(k+1|k)\}^T].$$

Whenever observation data at time t_{k+1} are available, a posteriori estimate $\hat{\mathbf{x}}(k+1|k+1)$ and its covariance matrix $\mathbf{P}(k+1|k+1)$, which should better represent the model under study, can be estimated in this way:

$$\hat{\mathbf{x}}(k+1|k+1) = [\mathbf{x}(k+1)|\mathbf{y}(1), \mathbf{y}(2), \ldots, \mathbf{y}(k+1)],$$
$$\mathbf{P}(k+1|k+1) = E[\{\mathbf{x}(k+1) - \hat{\mathbf{x}}(k+1|k+1)\}\{\mathbf{x}(k+1) - \hat{\mathbf{x}}(k+1|k+1)\}^T].$$

The EKF estimates a posteriori mean and covariance based on the recursive least squares. Initially, the EKF is assigned with the best *a priori* knowledge of the considered model, which can be obtained by *in situ* tests or laboratory experiments, as follows:

$$\begin{aligned}\hat{\mathbf{x}}(0|0) &= E[\mathbf{x}(0)],\\ \mathbf{P}(0|0) &= E[\{\mathbf{x}(0) - \hat{\mathbf{x}}(0|0)\}\{\mathbf{x}(0) - \hat{\mathbf{x}}(0|0)\}^T].\end{aligned} \tag{4}$$

The initial state, which consists of a set of guessed model parameters, is chosen based on engineering experience or preliminary examination of the structure. The closer the initial parameter values to the true ones are, the better initialization the EKF will be achieved, and therefore the faster convergence. In case of a complete lack of knowledge about the model parameters, the initial covariance matrix is assigned arbitrarily large due to a lack of confidence in the initial choice of the state vector.

Before observation data of the model are available, the EKF propagates the mean and error covariance of the state through time. The time update equations of the mean and covariance of the state are calculated as

$$\begin{aligned}\hat{\mathbf{x}}(k+1|k) &= \hat{\mathbf{x}}(k|k),\\ \mathbf{P}(k+1|k) &= \mathbf{I}\mathbf{P}(k|k)\mathbf{I}^T + \mathbf{Q},\end{aligned} \tag{5}$$

where $\mathbf{I}$ is the state transition matrix which is identity because we formulate the state transition Eq. (1a) as stationary.

As soon as the observation data are available, measurement update of the state and covariance can be performed following the below equations:

$$\mathbf{K}(k+1) = \mathbf{P}(k+1|k)\mathbf{H}^T(k+1)\{\mathbf{H}(k+1)\mathbf{P}(k+1|k)\mathbf{H}^T(k+1) + \mathbf{R}\}^{-1}, \tag{6a}$$

$$\hat{\mathbf{x}}(k+1|k+1) = \hat{\mathbf{x}}(k+1|k) + \mathbf{K}(k+1)\{\mathrm{y}_{\mathrm{exp}} - \mathbf{h}(\hat{\mathbf{x}}(k+1|k))\}, \tag{6b}$$

$$\mathbf{P}(k+1|k+1) = \mathbf{P}(k+1|k) - \mathbf{K}(k+1)\mathbf{H}(k+1)\mathbf{P}(k+1|k), \tag{6c}$$

where $\mathbf{H}$, which is termed sensitivity matrix, is composed of the derivatives of observation data with respect to the state variables. $\mathbf{H}$ cannot be analytically calculated in our problems because the observation (modeling) equation is not explicitly known. Instead, a numerical approximation of $\mathbf{H}$ will be performed using forward results from forward simulation (finite element analysis) as a black-box function $\mathbf{h}(\mathbf{x})$,

$$\mathbf{H}(k+1) = \frac{\partial \mathbf{h}(\mathbf{x}(k+1))}{\partial \mathbf{x}(k+1)}\Big|_{\mathbf{x}(k+1)=\hat{\mathbf{x}}(k+1|k)} \tag{7}$$